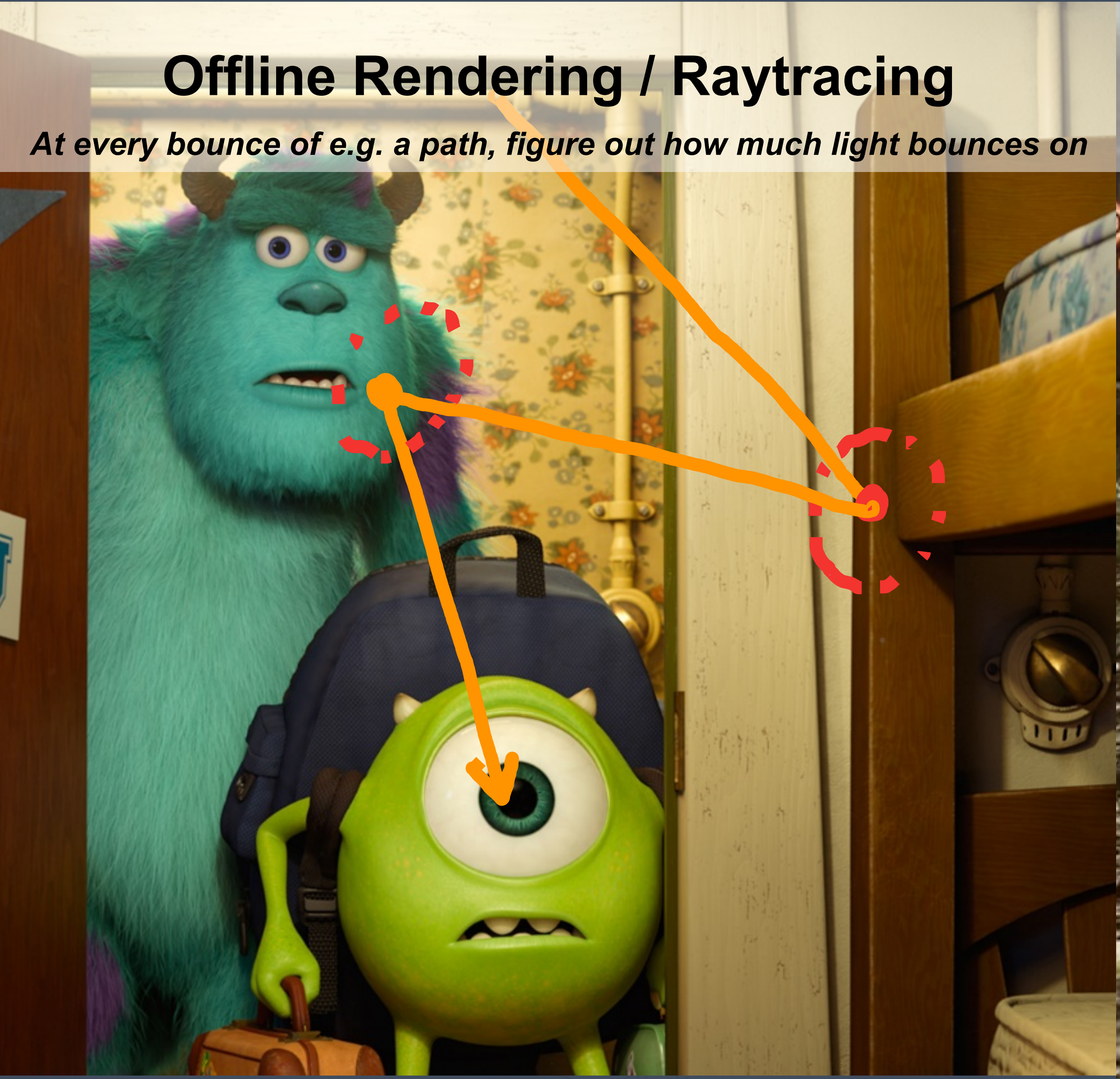
Advanced Shading

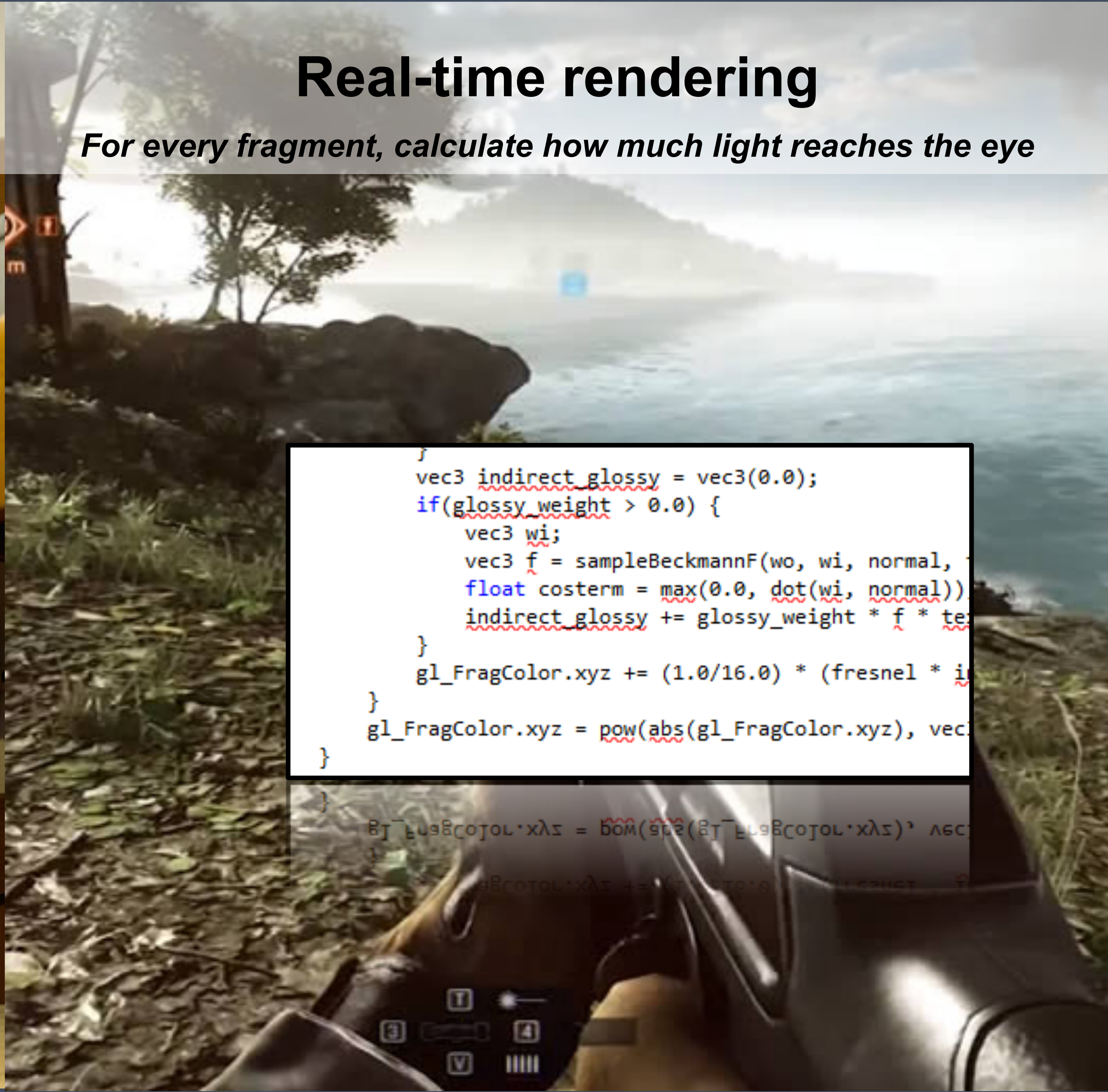
or: The Absolute Basics of Shading

What is Shading



Offline Rendering / Raytracing

At every bounce of e.g. a path, figure out how much light bounces on



Real-time rendering

For every fragment, calculate how much light reaches the eye

```
        vec3 indirect_glossy = vec3(0.0);
        if(glossy_weight > 0.0) {
            vec3 wi;
            vec3 f = sampleBeckmannF(wo, wi, normal,
            float costerm = max(0.0, dot(wi, normal))
            indirect_glossy += glossy_weight * f * te
        }
        gl_FragColor.xyz += (1.0/16.0) * (fresnel * i
    }
    gl_FragColor.xyz = pow(abs(gl_FragColor.xyz), vec
}
```

Who cares?

Everyone

- Almost all commercial renderers these days aim to be physically correct
    - Previously, laws were broken excessively to reduce noise and rendering times
    - Now laws are broken in a much more controlled manner
- Almost all game engines these days aim to use physically based materials, lights etc.
    - On older hardware, “correct” shading was impossible/too expensive.
    - With only direct lighting, didn’t really matter
    - Today engines incorporate lots of indirect lighting and the benefits of being physically based is becoming quite obvious.
- Realistic Images look realistic “for free”. Much less tweaking by artists.
- Physically based materials look “right” under any kind of lighting.
- When things look wrong, we can reason about why.

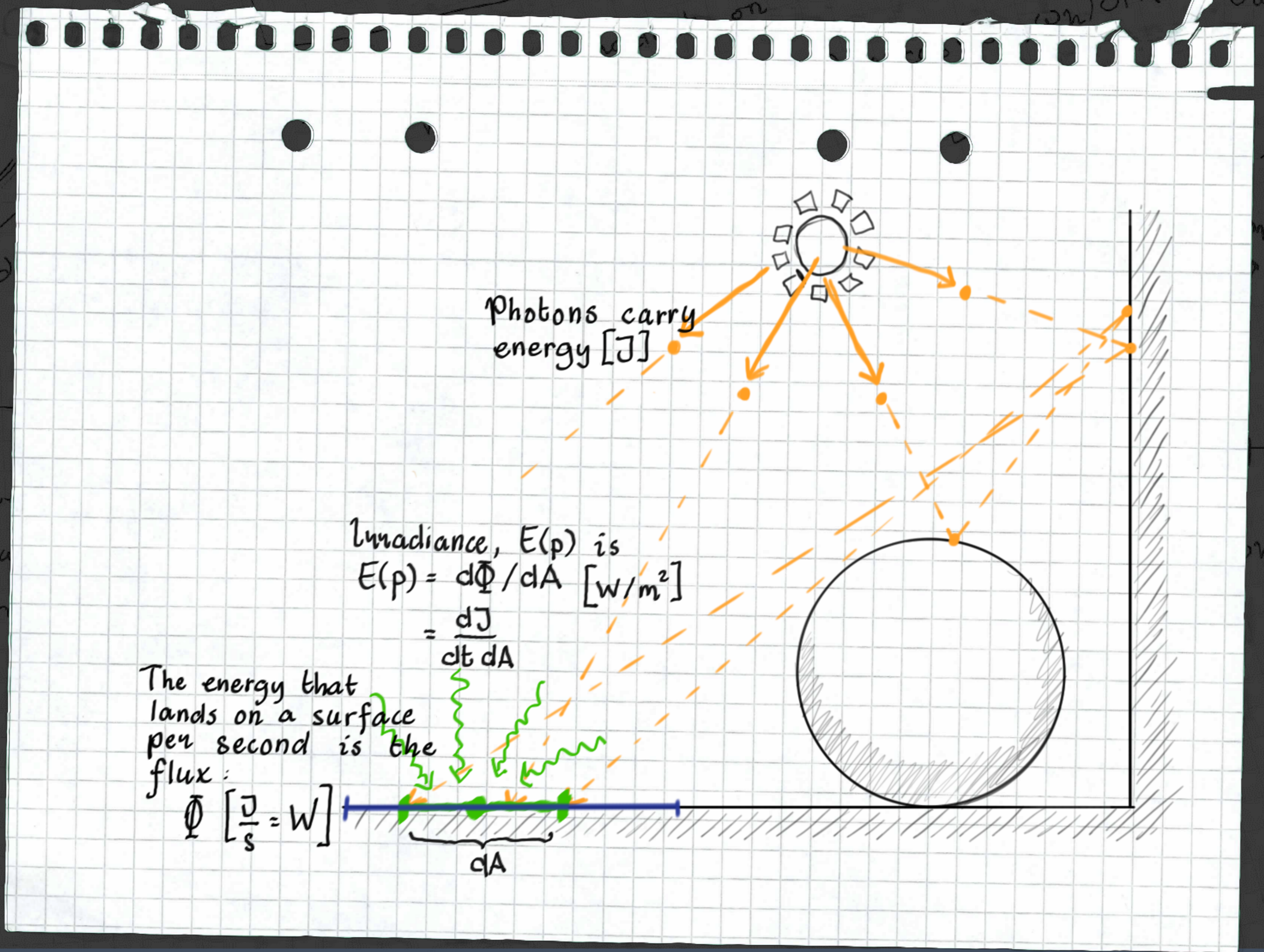
Why should *you* care?

- Implementing almost *any* CG algorithm is much easier when you know the underlying theory.
- In the (Game/CGI) industry, you are more than likely to have to write a baking tool, new material shader or GI algorithm some day.
- Most importantly: It’s fun stuff!

Basics: Radiometry

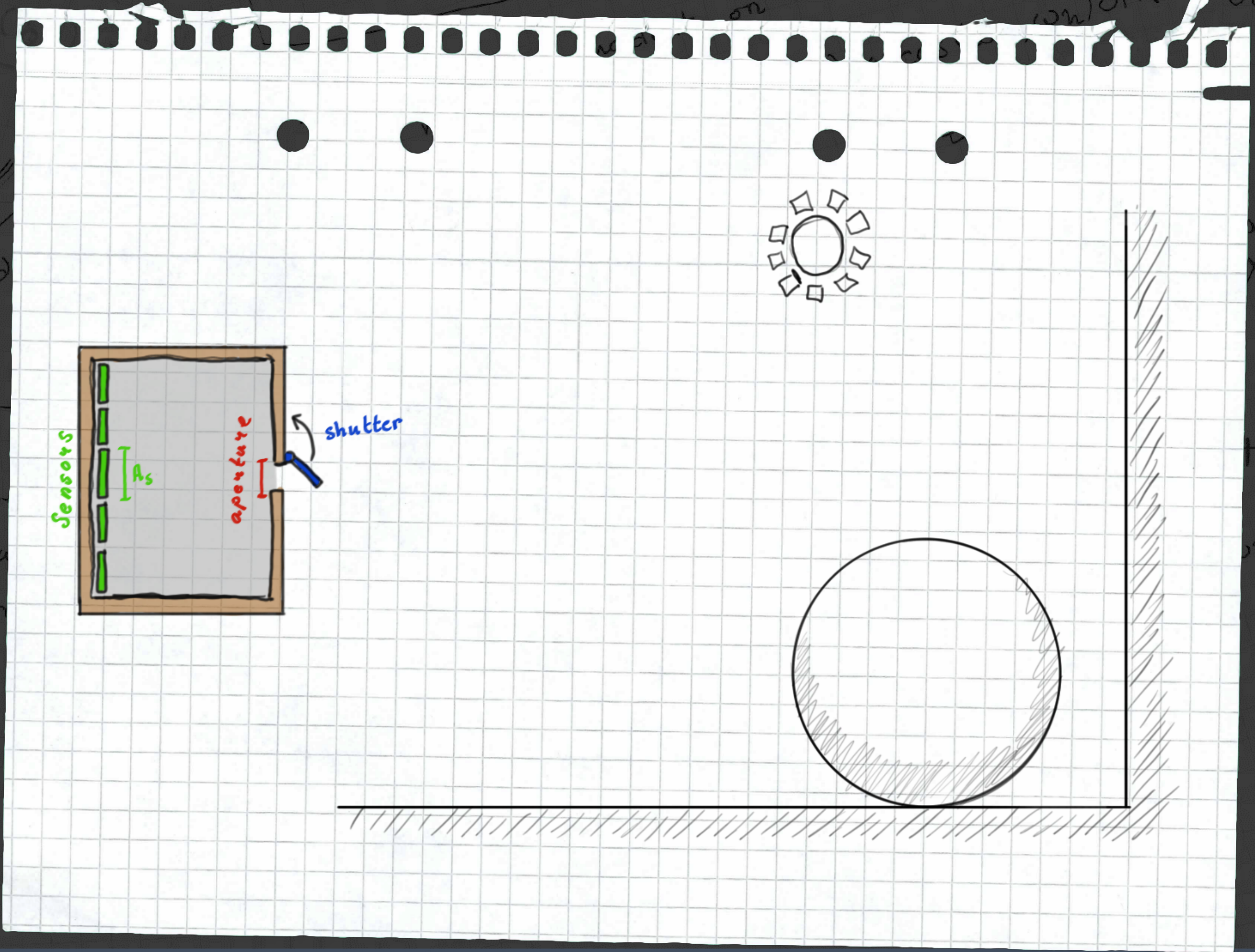
- Lightsource emits *photons.* Photons carry energy [J]
  (At a specific *wavelength*)
- The energy that passes a region of space per time unit is measured in radiant flux, $\Phi\ [J/s=W]$
- Irradiance (E) at some point p, is the *flux* per unit *area*.

$E(\mathbf{p})=d\Phi/dA\ [W/m\uparrow 2\ \ ]$



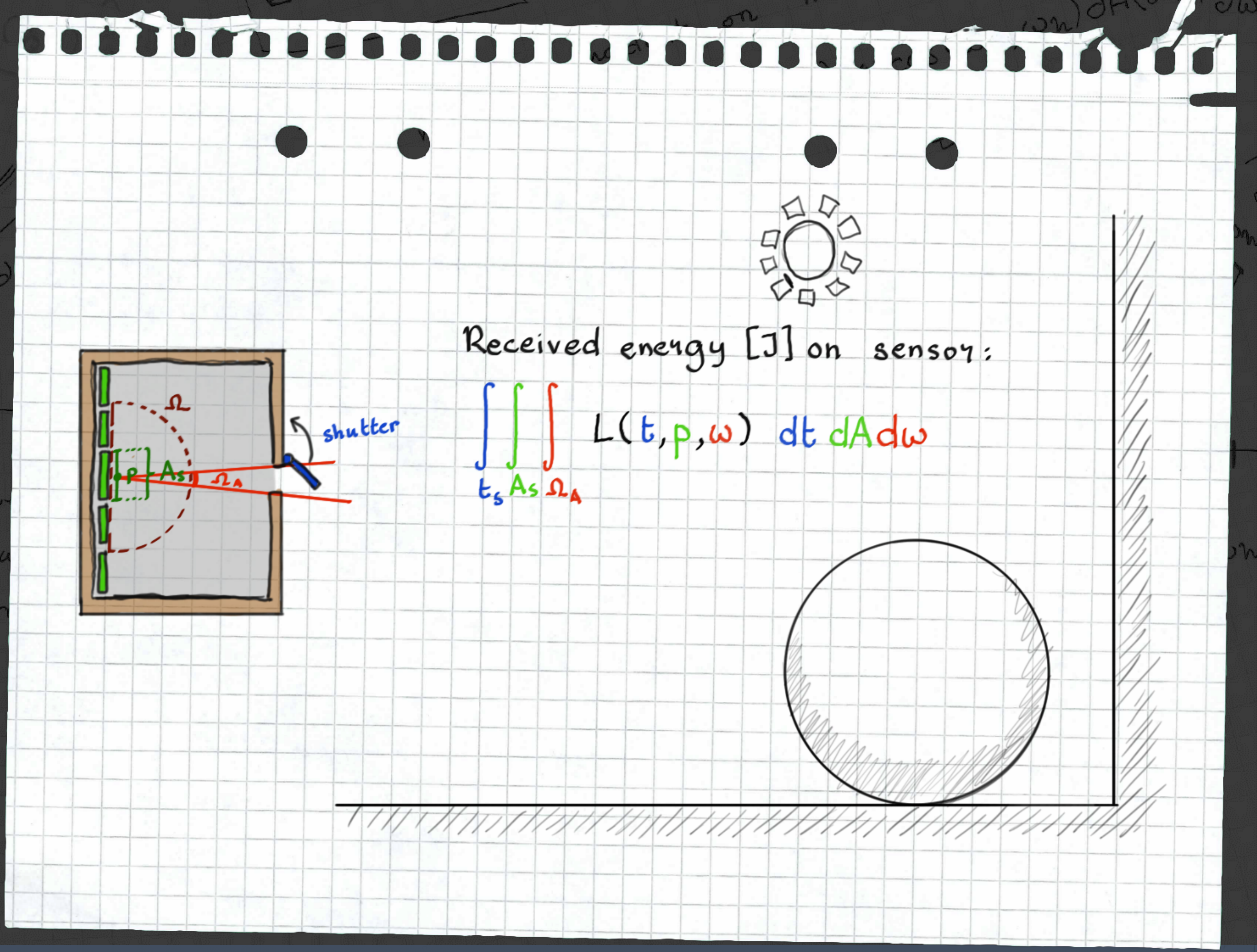
Basics: How an image is formed

- Let's consider a digital pinhole camera
  - The *shutter* opens to let in photons for a short time (t_s)
  - The aperture is the size of the little pinhole
  - Each sensor, with area A_s will measure the energy (number of photons) that hit it during t_s



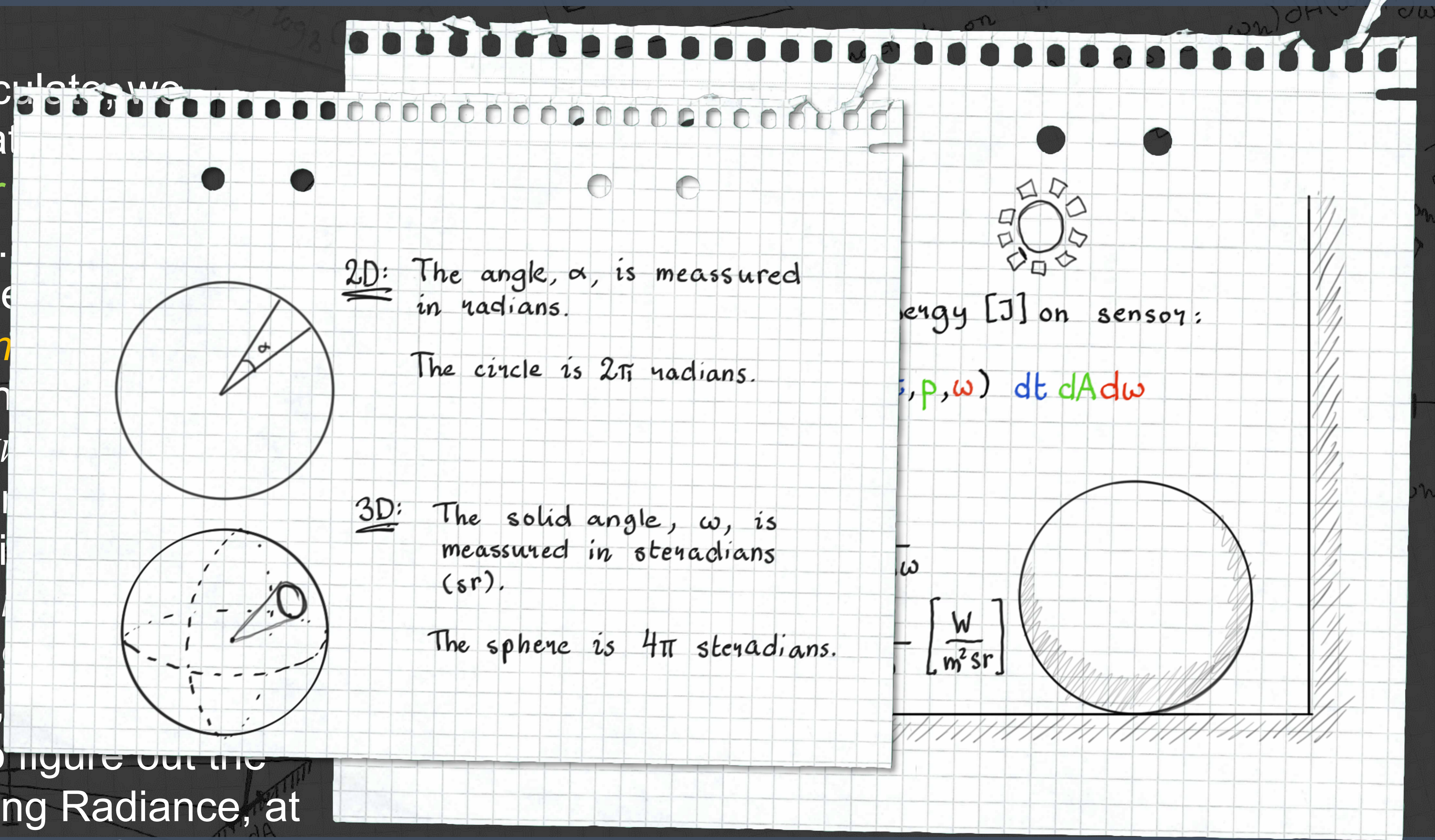
Basics: How an image is formed

- Let’s consider a digital pinhole camera
  - To calculate, we integrate over time, sensor area and angle



Basics: How an image is formed

- To calc ate, we
  integrat
  sensor
  angle..
  And me
  Radian
- Radian
  $L(\boldsymbol{p},\omega)$ [V
  is the i
  per uni
  solid a
- So, to
  image,
  able to figure out the
  incoming Radiance, at
  some point, from



2D: The angle, α, is meassured in radians.

The circle is 2π radians.

3D: The solid angle, ω, is measured in steradians (sr).

The sphere is 4π steradians.

ergy [J] on sensor:

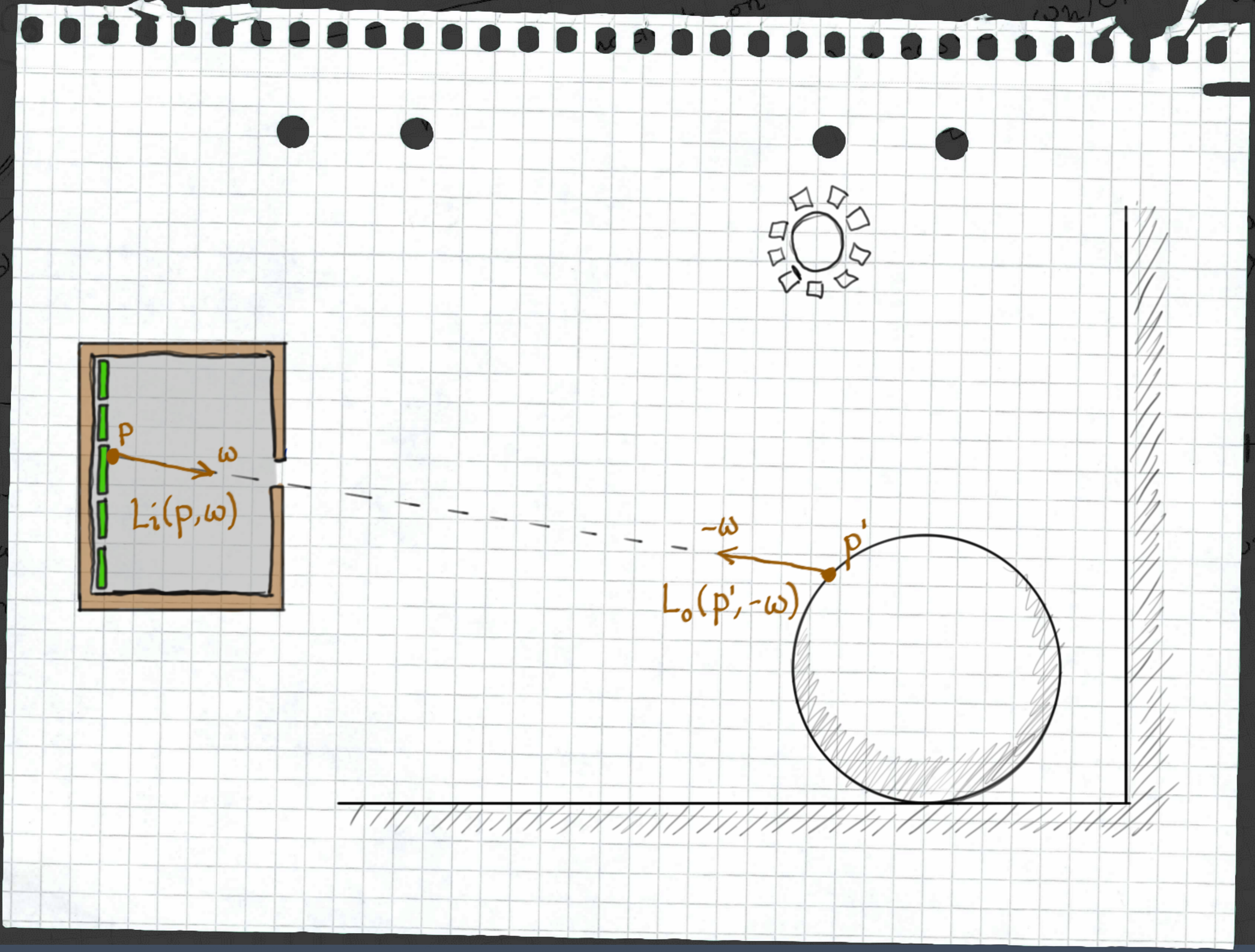
$,p,\omega)\ dt\ dA\ d\omega$

ω

$\left[\frac{W}{m^2 sr}\right]$

Basics: How an image is formed

- Radiance does not change along a ray between two surfaces!
- So to figure out the incoming radiance from ω at **p**, we shoot a ray to find **p'** and figure out the *outgoing* radiance in direction -ω

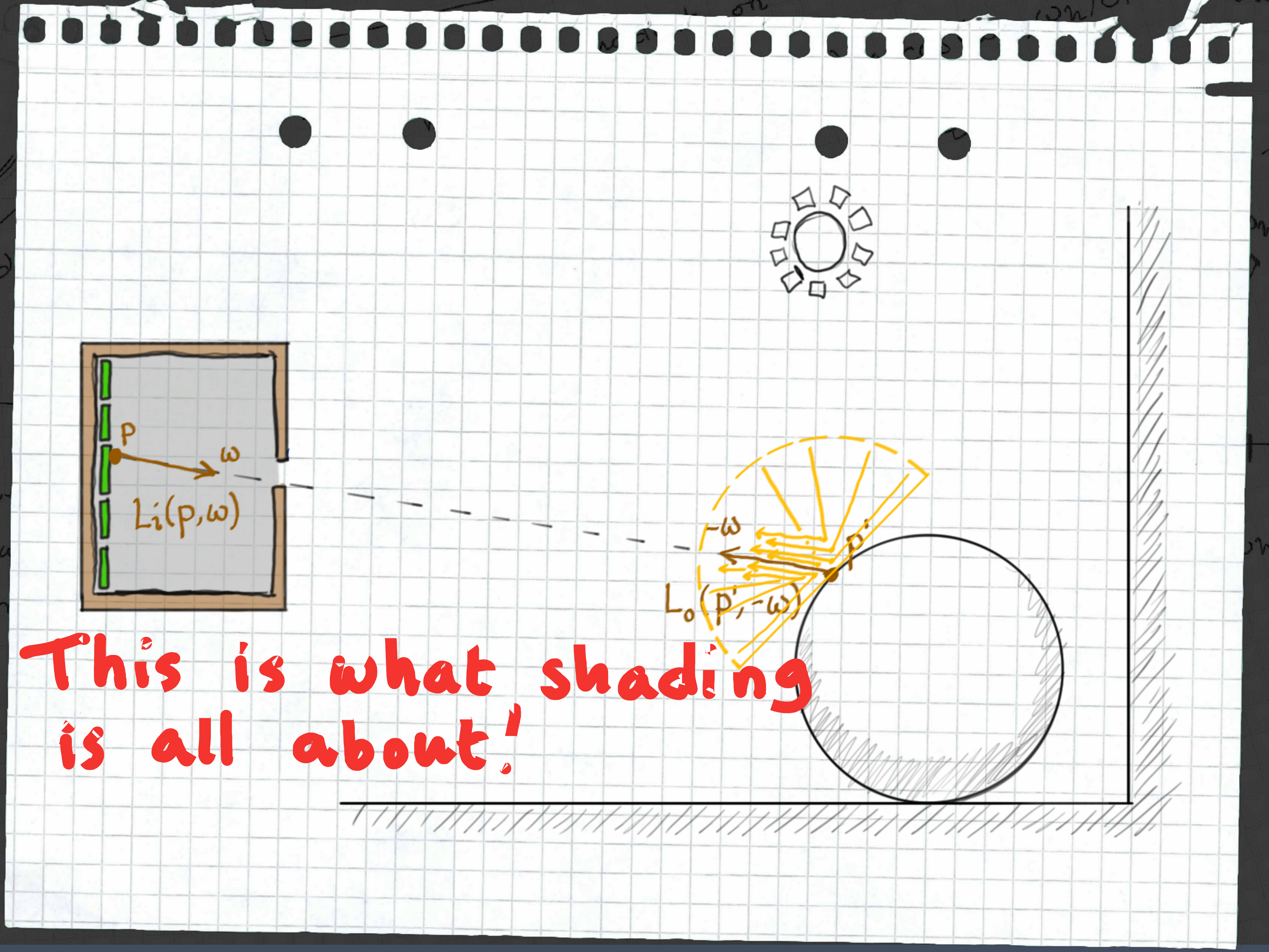


Basics: How an image is formed

- Radiance does not change along a ray between two surfaces!

- So to figure out the incoming radiance from ω at **p**, we shoot a ray to find [illegible] the *outgoing* radiance in direction –ω

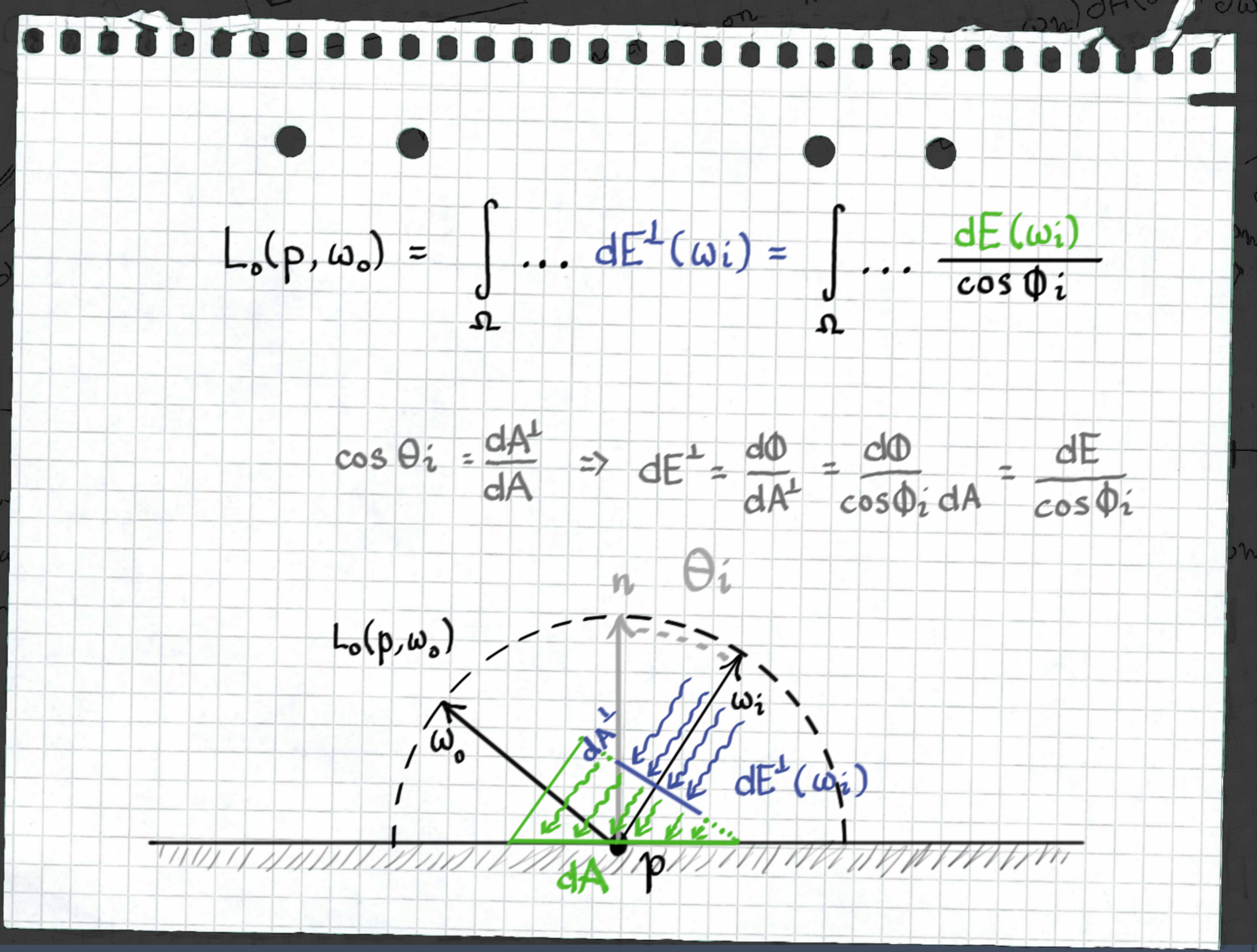
- To find $L{\downarrow}o\,(\boldsymbol{p},-\omega)$ we must consider all *incoming* radi[illegible]cide how much of it reflects towards $-\omega$.



This is what shading is all about!

Surface Reflection

- We wan't to figure out the *ourgoing* radiance for some direction at a point p.
- We integrate over the hemisphere
- For every incoming direction, we can find out the differential irradiance over $dA\uparrow\perp$
- We *want* to know the irradiance on the surface, dA

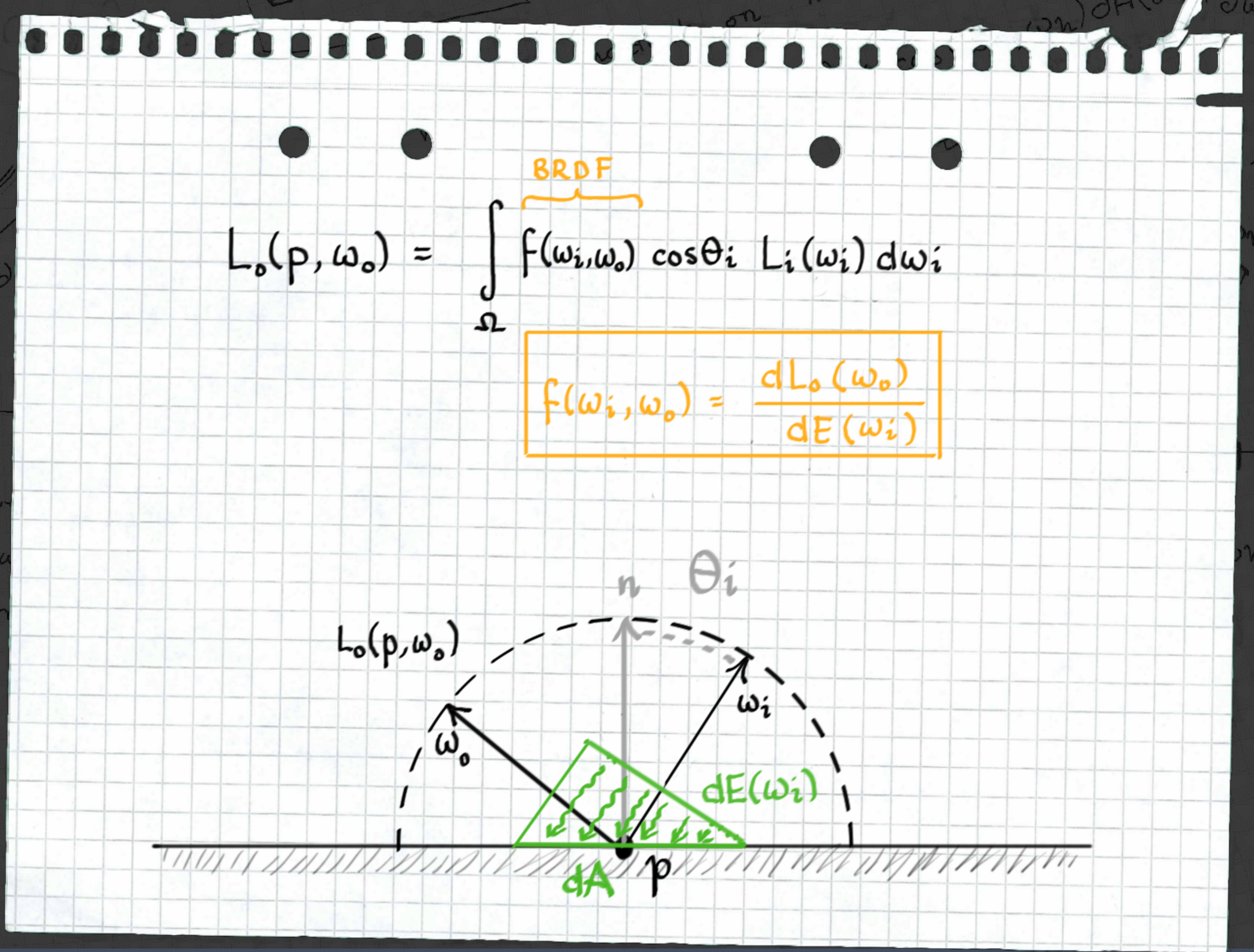


$$L_o(p,\omega_o) = \int_\Omega \dots dE^\perp(\omega_i) = \int_\Omega \dots \frac{dE(\omega_i)}{\cos\Phi_i}$$

$$\cos\Theta_i = \frac{dA^\perp}{dA} \Rightarrow dE^\perp = \frac{d\Phi}{dA^\perp} = \frac{d\Phi}{\cos\Phi_i\, dA} = \frac{dE}{\cos\Phi_i}$$

Surface Reflection

- What we want to sum is *radiance*. To get there, we make an assumption.
- We invent a function that for any incoming and outgoing directions converts from differential irradiance to differential outgoing radiance.
- This function is the BRDF

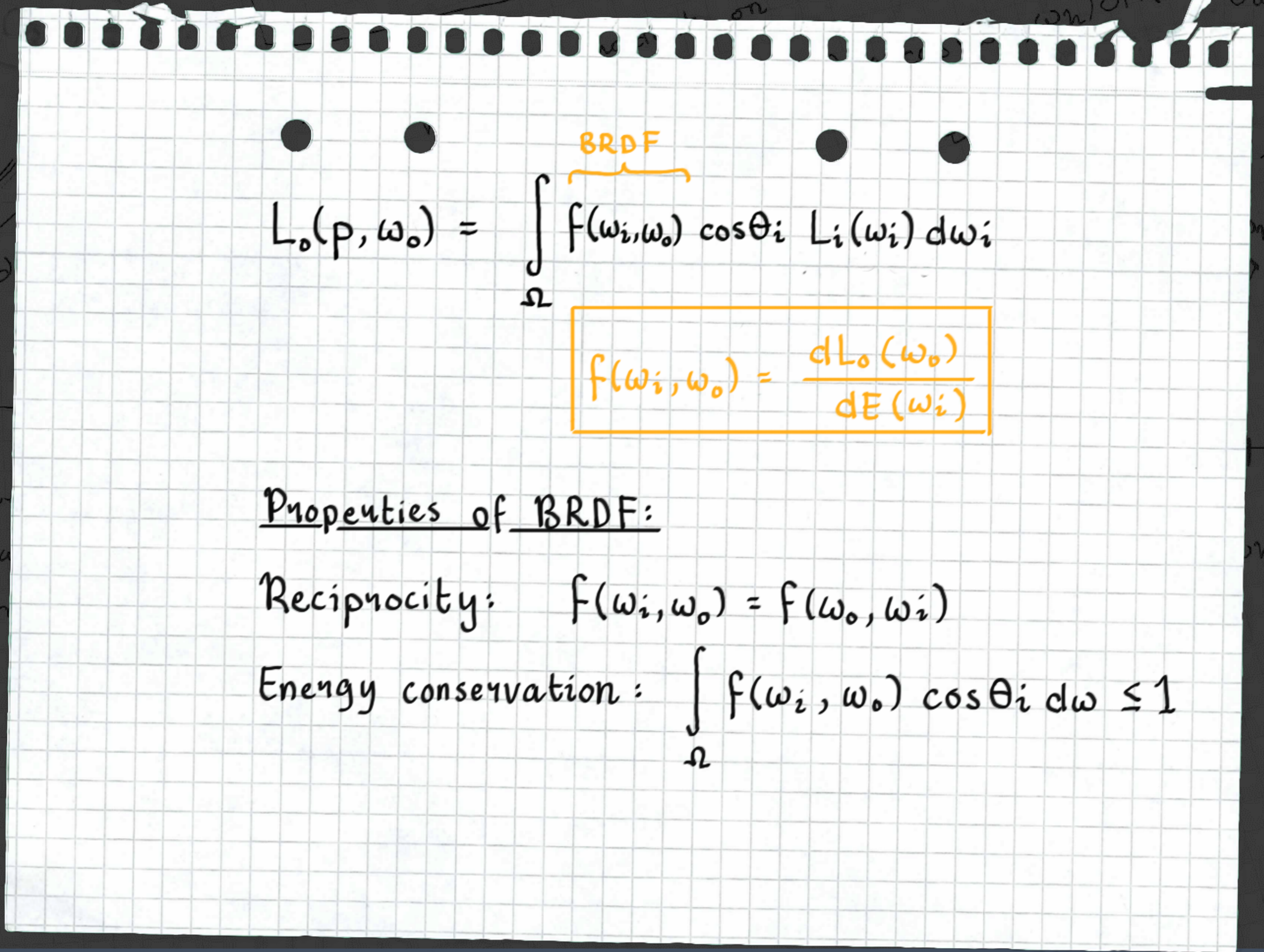


$$L_o(p,\omega_o) = \int_\Omega \overbrace{f(\omega_i,\omega_o)}^{\text{BRDF}} \cos\theta_i \; L_i(\omega_i)\, d\omega_i$$

$$\boxed{f(\omega_i,\omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)}}$$

The BRDF

- The BRDF describes the material
- A physically based BRDF has some important properties



$$L_o(p,\omega_o) = \int_\Omega \overbrace{f(\omega_i,\omega_o)}^{\text{BRDF}} \cos\theta_i \; L_i(\omega_i)\, d\omega_i$$

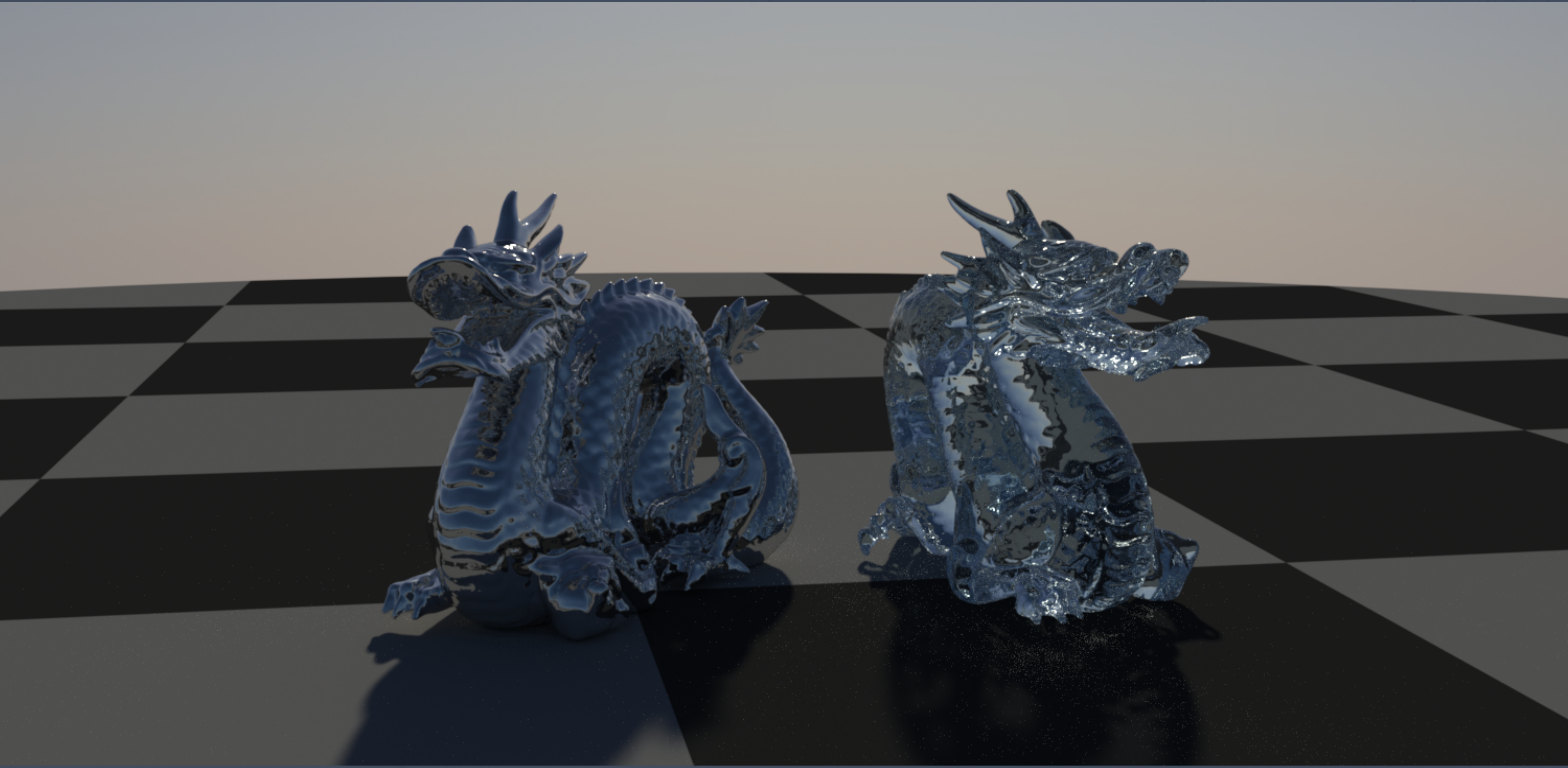
$$\boxed{f(\omega_i,\omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)}}$$

Properties of BRDF:

Reciprocity: $f(\omega_i,\omega_o) = f(\omega_o,\omega_i)$

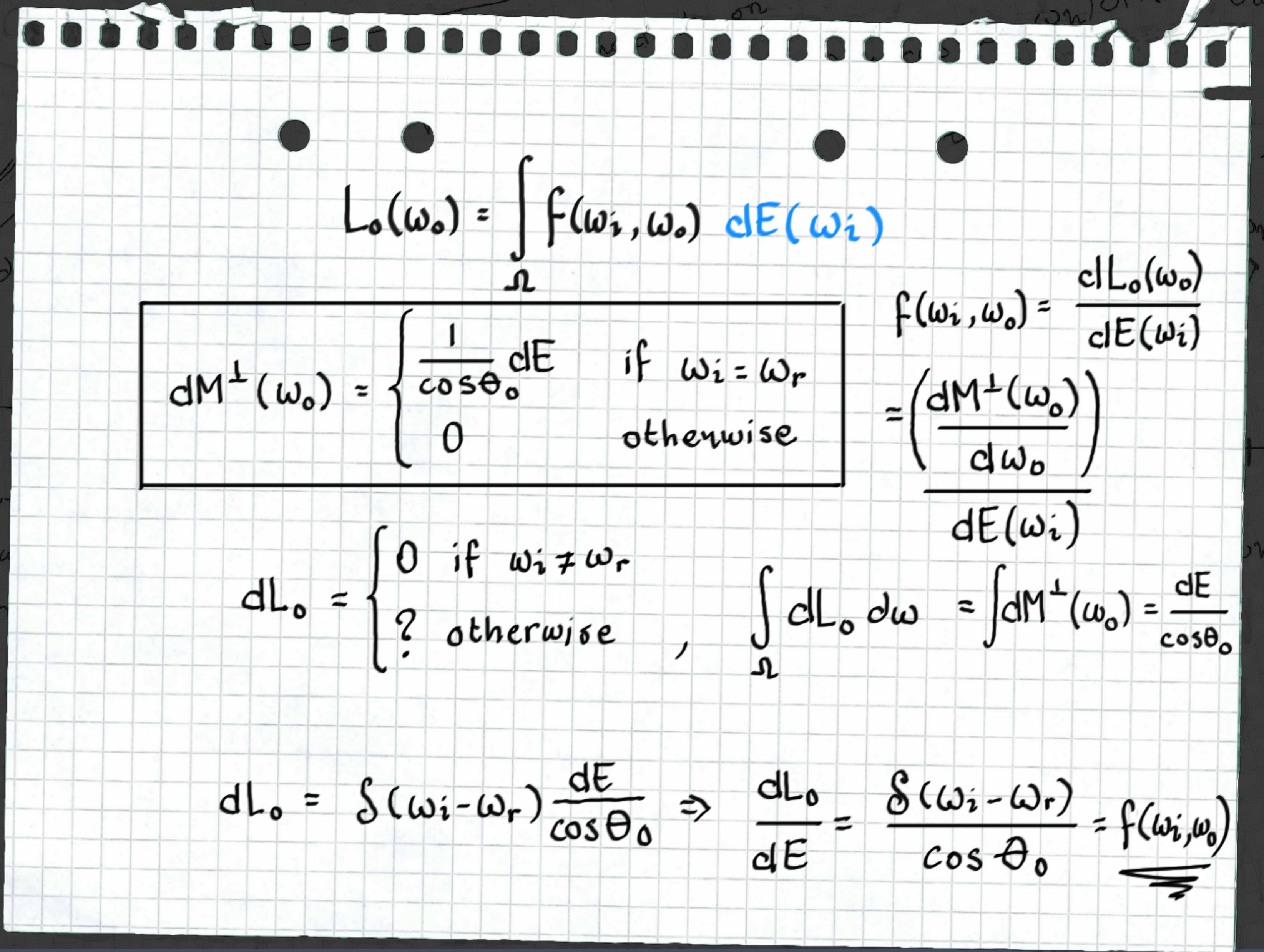
Energy conservation: $\int_\Omega f(\omega_i,\omega_o)\cos\theta_i \, d\omega \leq 1$

Optically smooth materials



Ideal specular reflection

- All energy incoming from $\omega\downarrow i$ shall be reflected in $\omega\downarrow r$
- So, the differential *Radiant Exitance (or Radiosity)* over a surface perpendicular to $\omega\downarrow o$ is easily obtained.
- But we can't find derivative!
- We know though, that it's another spikey function and that if we integrate over all outgoing directions we get the radiosity.



$$L_o(\omega_o) = \int_\Omega f(\omega_i,\omega_o)\, dE(\omega_i)$$

$$\boxed{dM^\perp(\omega_o) = \begin{cases}\frac{1}{\cos\theta_o}dE & \text{if } \omega_i=\omega_r\\ 0 & \text{otherwise}\end{cases}}$$

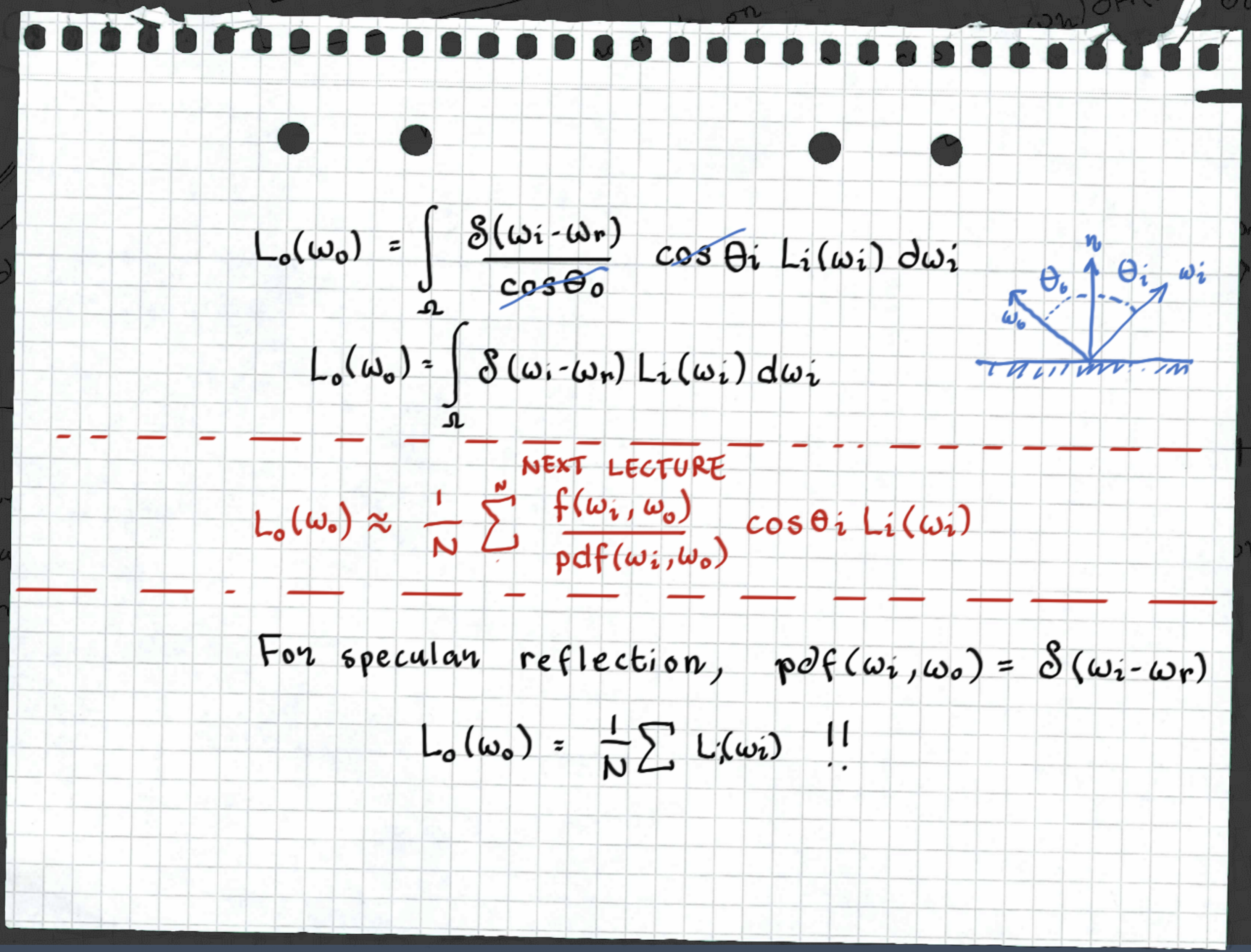
$$f(\omega_i,\omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)} = \frac{\left(\frac{dM^\perp(\omega_o)}{d\omega_o}\right)}{dE(\omega_i)}$$

$$dL_o = \begin{cases}0 & \text{if } \omega_i\neq\omega_r\\ ? & \text{otherwise}\end{cases}, \quad \int_\Omega dL_o\, d\omega = \int dM^\perp(\omega_o) = \frac{dE}{\cos\theta_o}$$

$$dL_o = \delta(\omega_i-\omega_r)\frac{dE}{\cos\theta_o} \Rightarrow \frac{dL_o}{dE} = \frac{\delta(\omega_i-\omega_r)}{\cos\theta_o} = \underline{\underline{f(\omega_i,\omega_o)}}$$

Ideal specular reflection

- "But I've implemented ideal specular reflection and it is nowhere *near* this complicated!"



$$L_o(\omega_o) = \int_\Omega \frac{\delta(\omega_i-\omega_r)}{\cancel{\cos\theta_o}} \cancel{\cos}\,\theta_i\, L_i(\omega_i)\, d\omega_i$$

$$L_o(\omega_o) = \int_\Omega \delta(\omega_i-\omega_r)\, L_i(\omega_i)\, d\omega_i$$

NEXT LECTURE

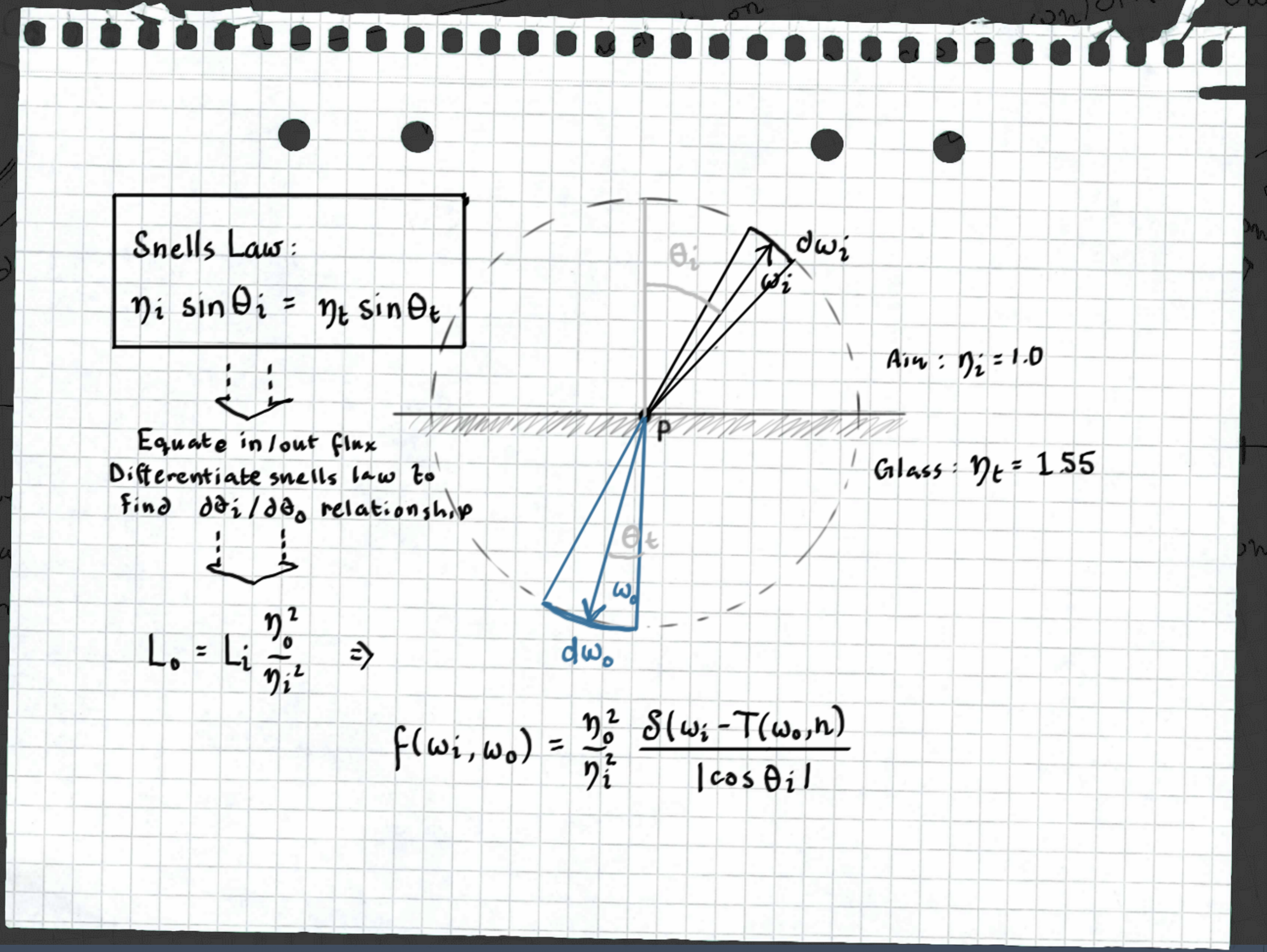
$$L_o(\omega_o) \approx \frac{1}{N}\sum^{N} \frac{f(\omega_i,\omega_o)}{pdf(\omega_i,\omega_o)} \cos\theta_i\, L_i(\omega_i)$$

For specular reflection, $pdf(\omega_i,\omega_o) = \delta(\omega_i-\omega_r)$

$$L_o(\omega_o) = \frac{1}{N}\sum L_i(\omega_i) \quad !!$$

Ideal specular refraction

- What about refraction?
- $d\omega\downarrow i \neq d\omega\downarrow o$
- Radiance changes as it passes from one media to another.



Snells Law:

$\eta_i \sin\theta_i = \eta_t \sin\theta_t$

Equate in/out flux

Differentiate snells law to find $\partial\theta_i / \partial\theta_o$ relationship

$L_o = L_i \frac{\eta_o^2}{\eta_i^2} \Rightarrow$

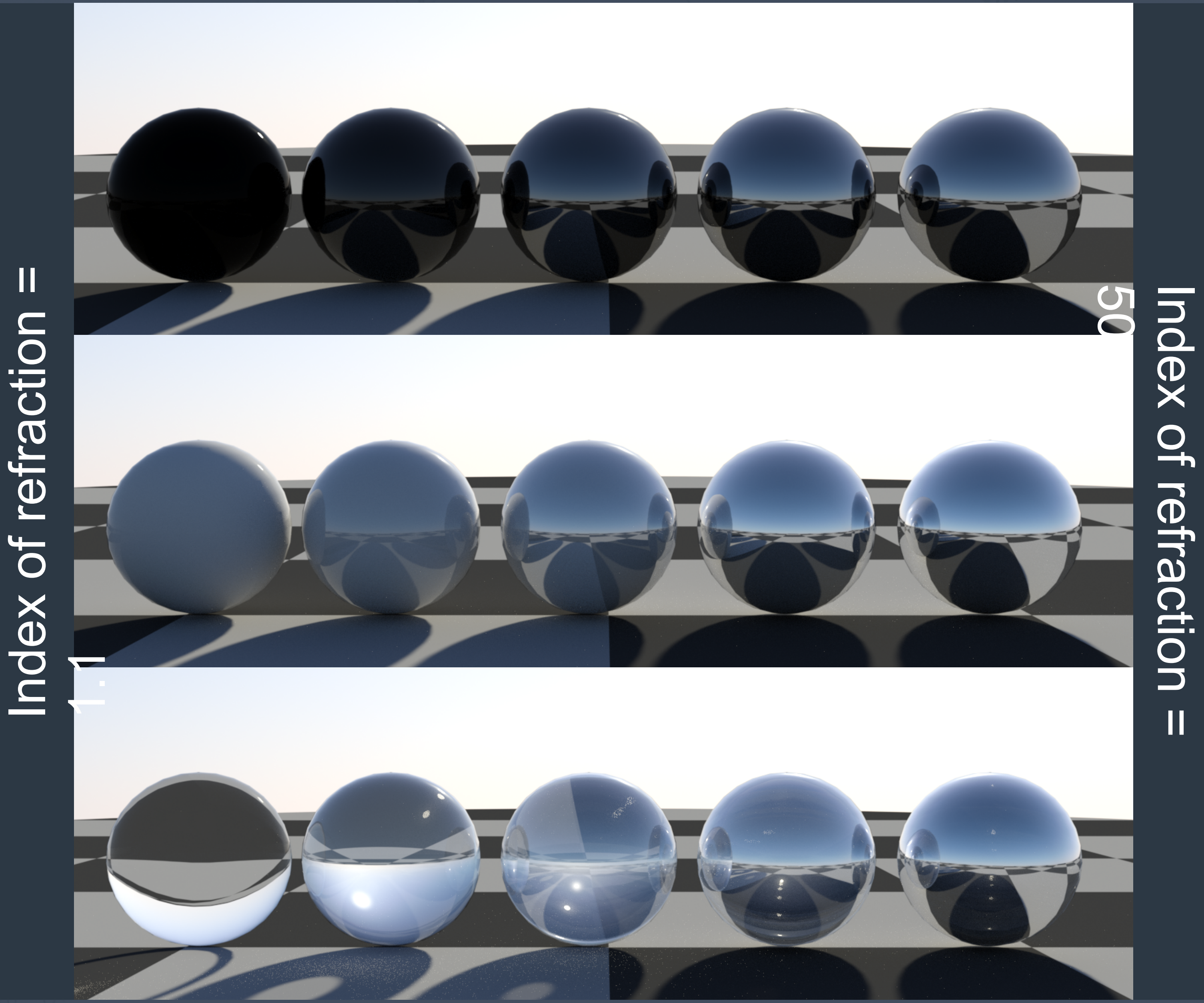
$f(\omega_i, \omega_o) = \frac{\eta_o^2}{\eta_i^2} \frac{\delta(\omega_i - T(\omega_o, n))}{|\cos\theta_i|}$

Fresnel

Conductor (e.g. metals)

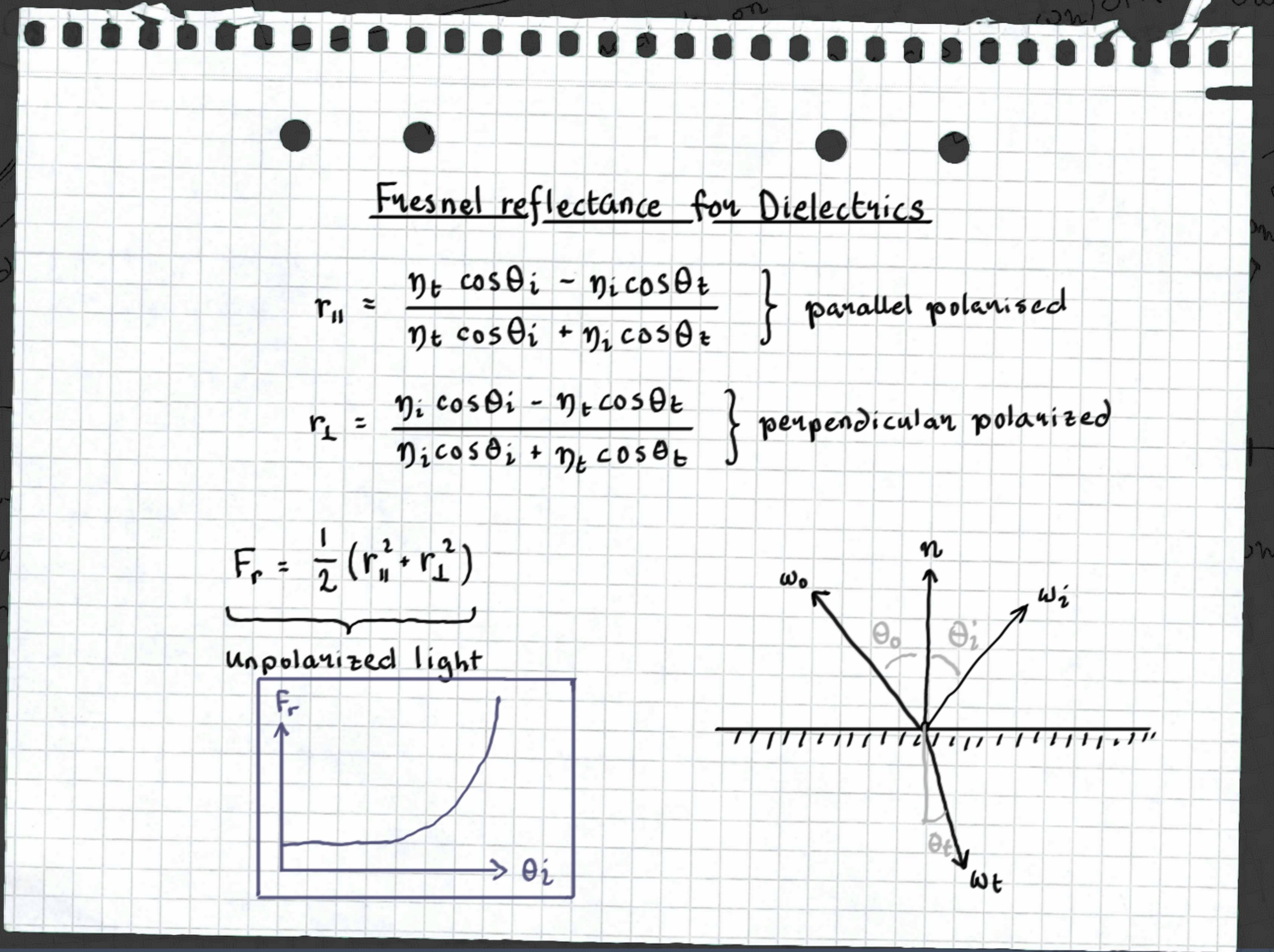
Non-transparent Dielectric (e.g. plastic)

Transparent Dielectric (e.g. glass)



Fresnel

- A close approximation for Dielectrics
- In Computer Graphics, we rarely care about polarization
- This effect is easily observed on e.g. a still lake



Fresnel reflectance for Dielectrics

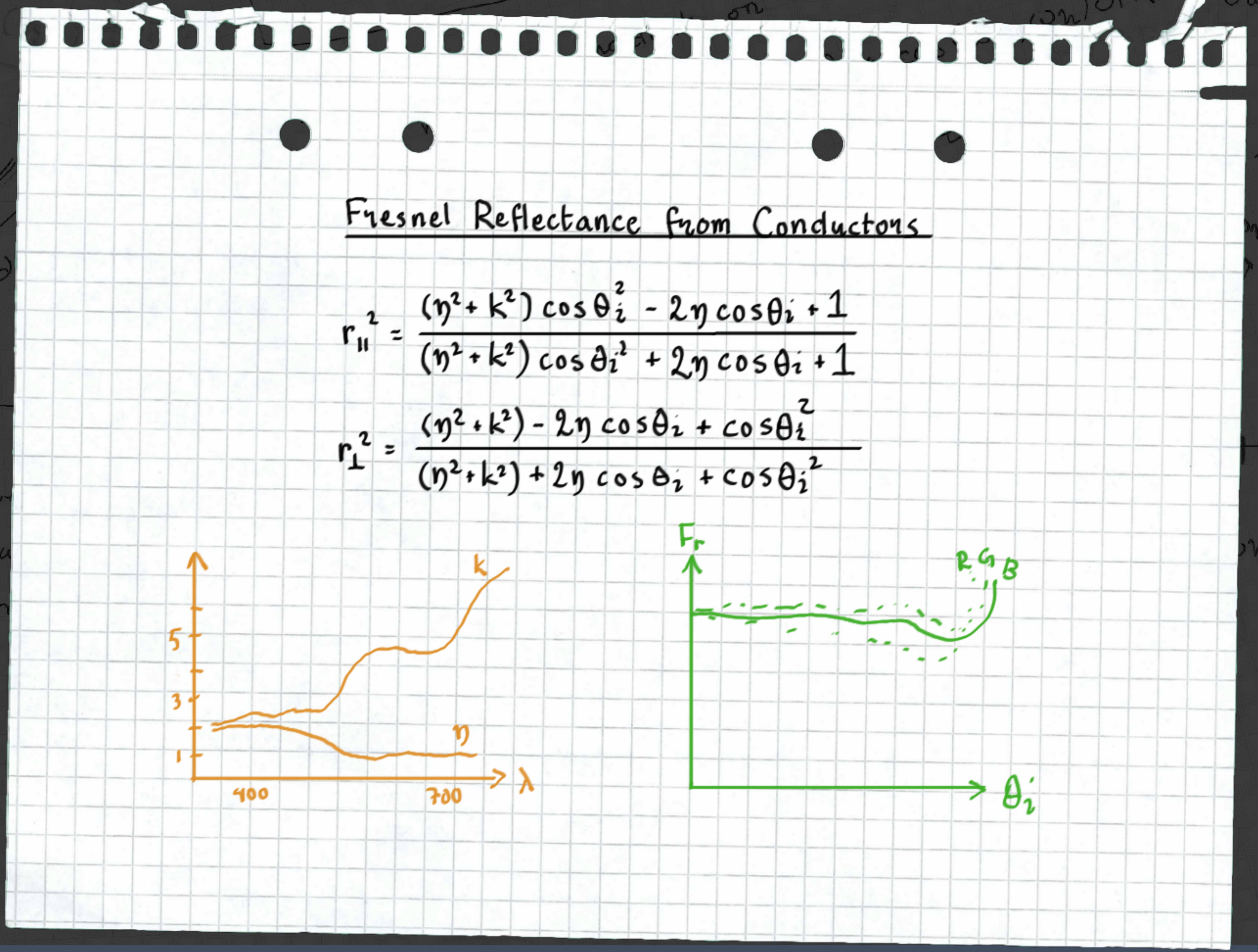
$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t} \quad \Big\} \text{ parallel polarised}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t} \quad \Big\} \text{ perpendicular polarized}$$

$$\underbrace{F_r = \frac{1}{2}\left(r_{\parallel}^2 + r_{\perp}^2\right)}_{\text{Unpolarized light}}$$

Fresnel

- Conductors don't refract, but energy is turned into heat.
- Equations depend on index of refraction *and* absorption coefficient
- Both are wavelength dependent



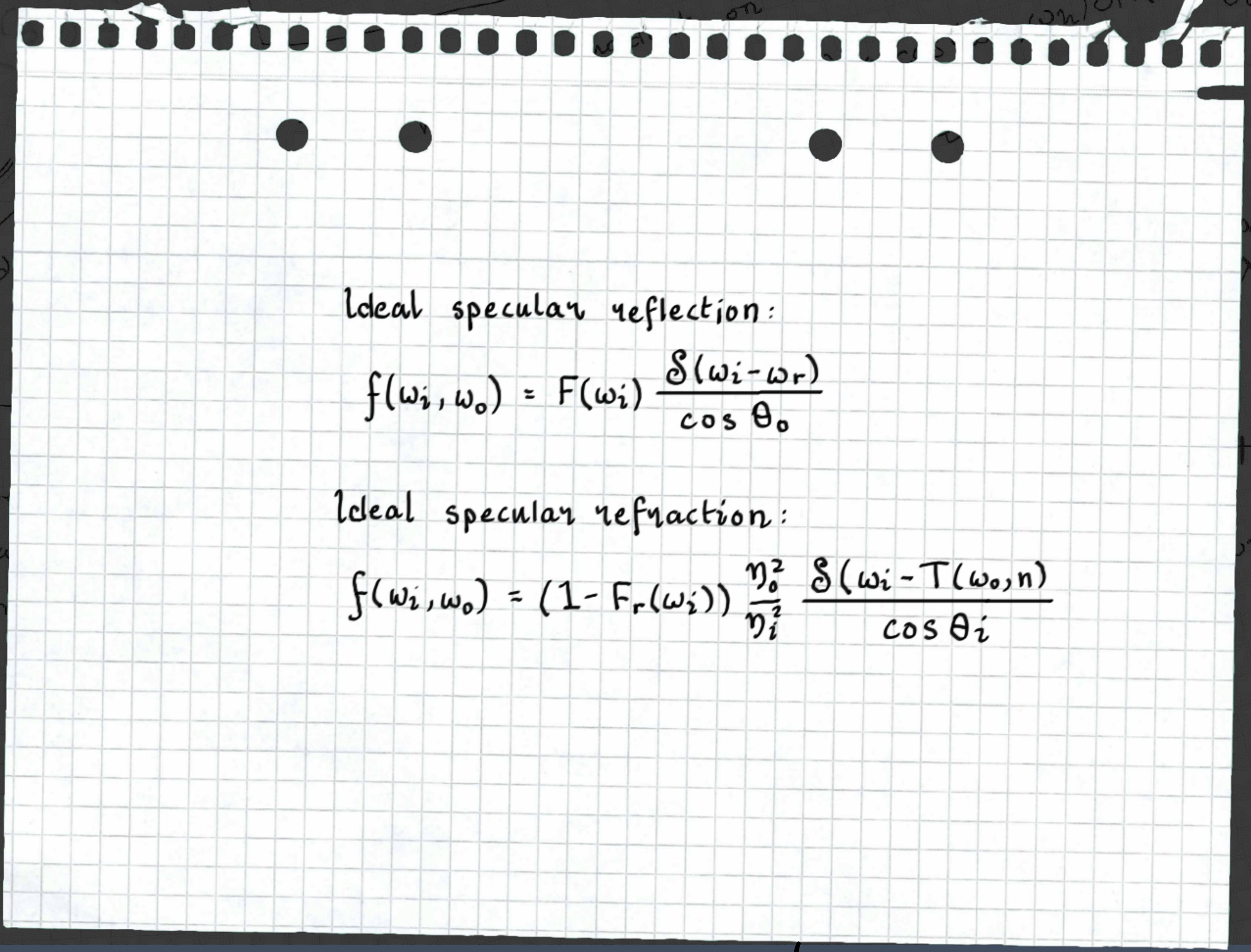
Fresnel Reflectance from Conductors

$$r_{\parallel}^2 = \frac{(\eta^2+k^2)\cos\theta_i^2 - 2\eta\cos\theta_i + 1}{(\eta^2+k^2)\cos\theta_i^2 + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2+k^2) - 2\eta\cos\theta_i + \cos\theta_i^2}{(\eta^2+k^2) + 2\eta\cos\theta_i + \cos\theta_i^2}$$

Fresnel

- So for physically based ideal smooth surfaces, the BRDF takes fresnel into account



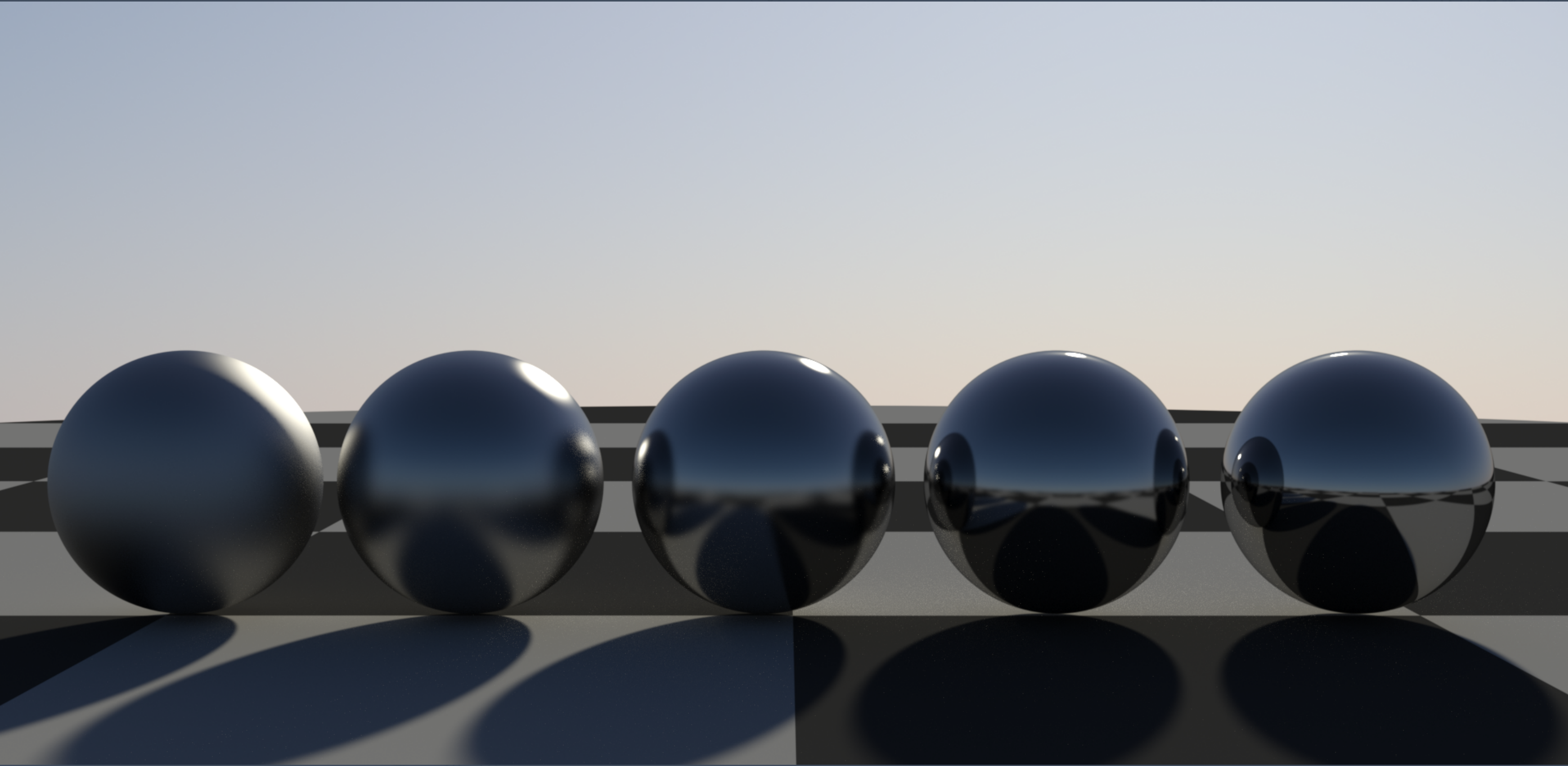
Ideal specular reflection:

$$f(\omega_i, \omega_o) = F(\omega_i) \frac{\delta(\omega_i - \omega_r)}{\cos \theta_o}$$

Ideal specular refraction:

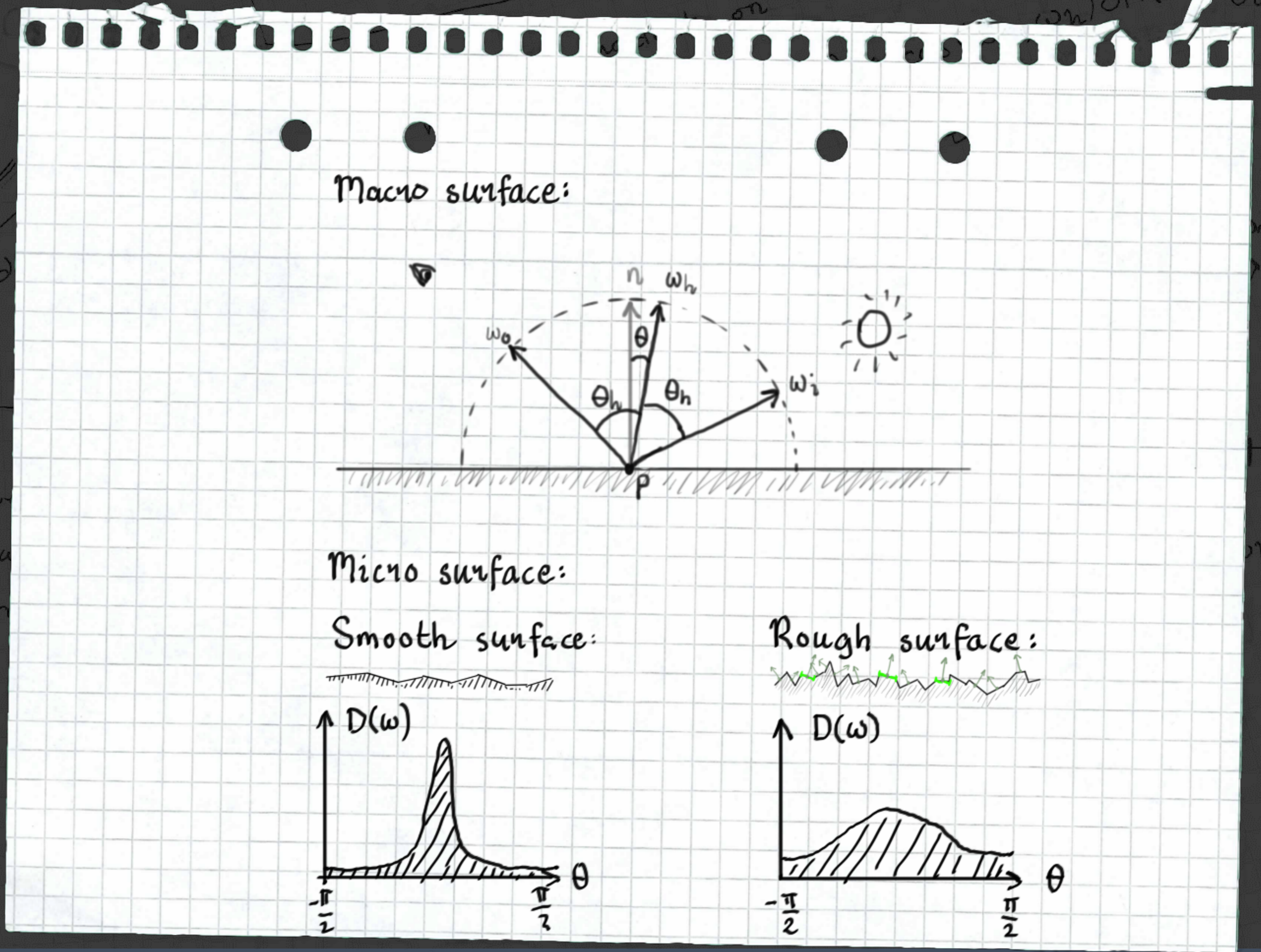
$$f(\omega_i, \omega_o) = (1 - F_r(\omega_i)) \frac{\eta_o^2}{\eta_i^2} \frac{\delta(\omega_i - T(\omega_o, n))}{\cos \theta_i}$$

Microfacet Theory



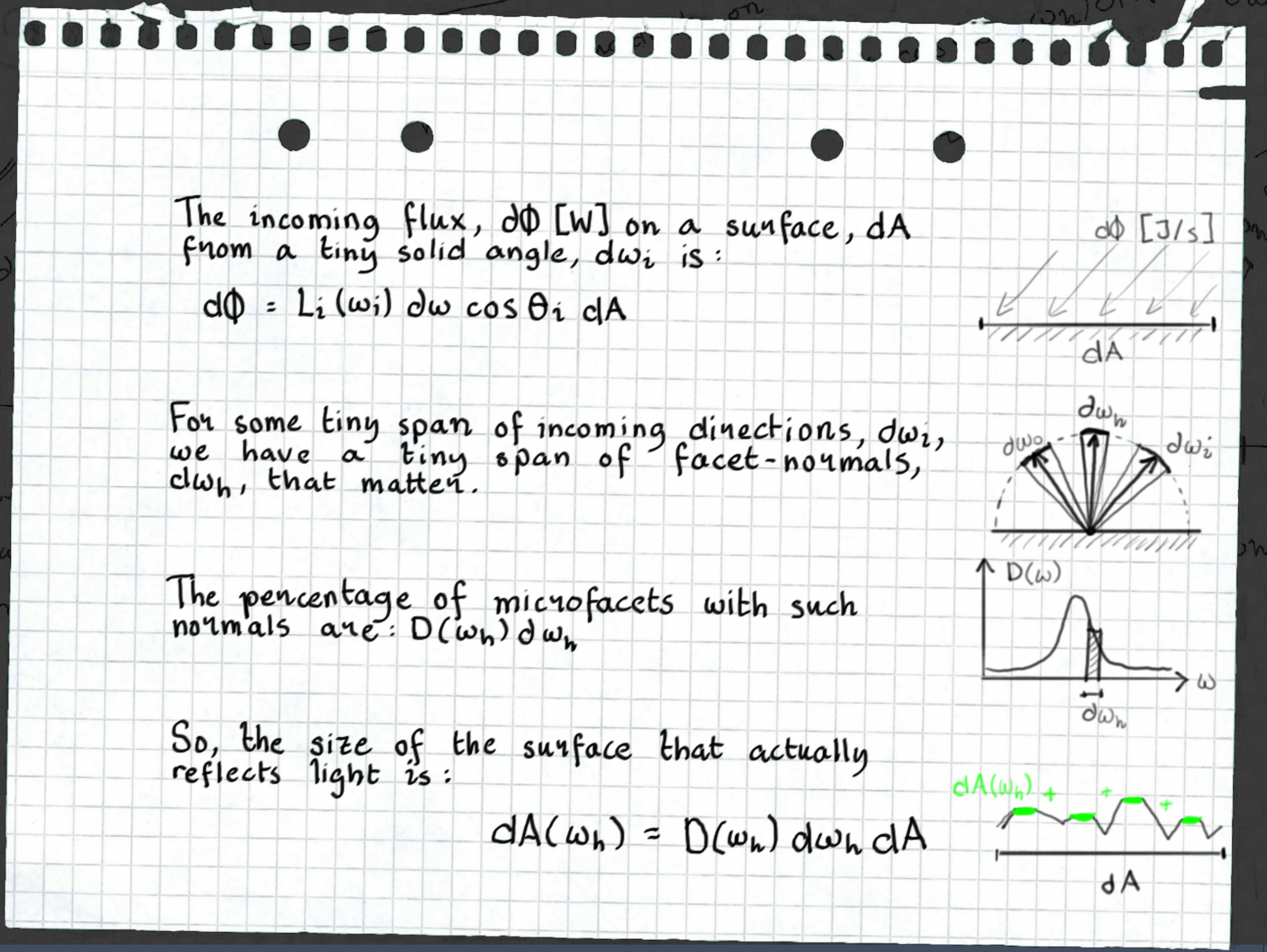
Microfacet theory

- Torrance and Sparrow [1967] suggested that rough surfaces can be modelled as collections of ideally smooth mirrors
- We will then only see reflections from those microsurfaces that have normal = half-angle
- The surface is described by a distribution function D(w), which says how likely a microsurface is to have normal = w



Microfacet theory

- We begin by figuring out how much flux is reflected from one direction.



The incoming flux, $d\Phi$ [W] on a surface, dA from a tiny solid angle, $d\omega_i$ is:

$$d\Phi = L_i(\omega_i)\, d\omega \cos\theta_i\, dA$$

For some tiny span of incoming directions, $d\omega_i$, we have a tiny span of facet-normals, $d\omega_h$, that matter.

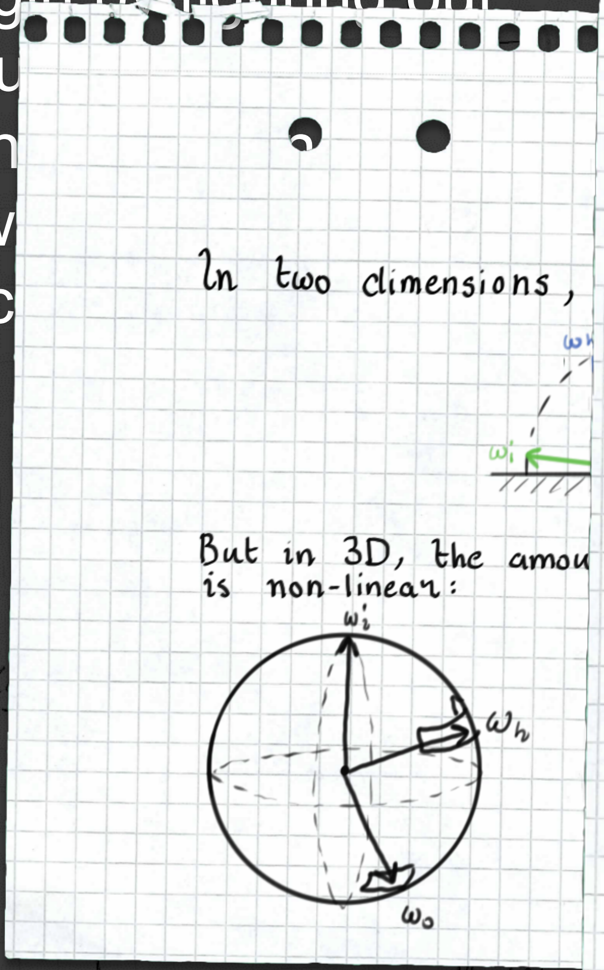
The percentage of microfacets with such normals are: $D(\omega_h)\, d\omega_h$

So, the size of the surface that actually reflects light is:

$$dA(\omega_h) = D(\omega_h)\, d\omega_h\, dA$$

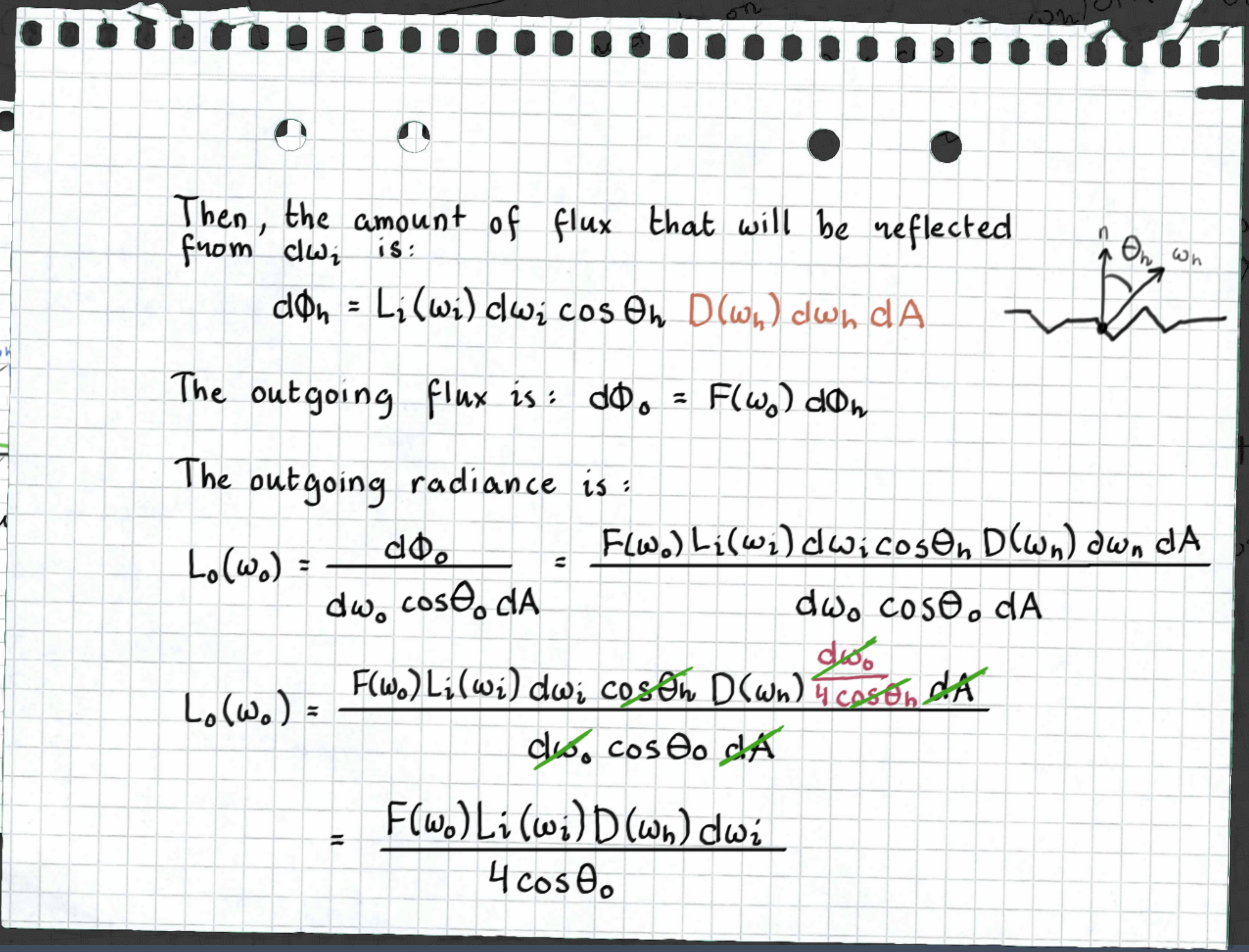
Microfacet theory

- We begin by figuring out how mu from on
- Then w radianc



In two dimensions,

But in 3D, the amou is non-linear:



Then, the amount of flux that will be reflected from $d\omega_i$ is:

$$d\Phi_h = L_i(\omega_i)\,d\omega_i \cos\theta_h\, D(\omega_h)\, d\omega_h\, dA$$

The outgoing flux is: $d\Phi_o = F(\omega_o)\,d\Phi_h$

The outgoing radiance is:

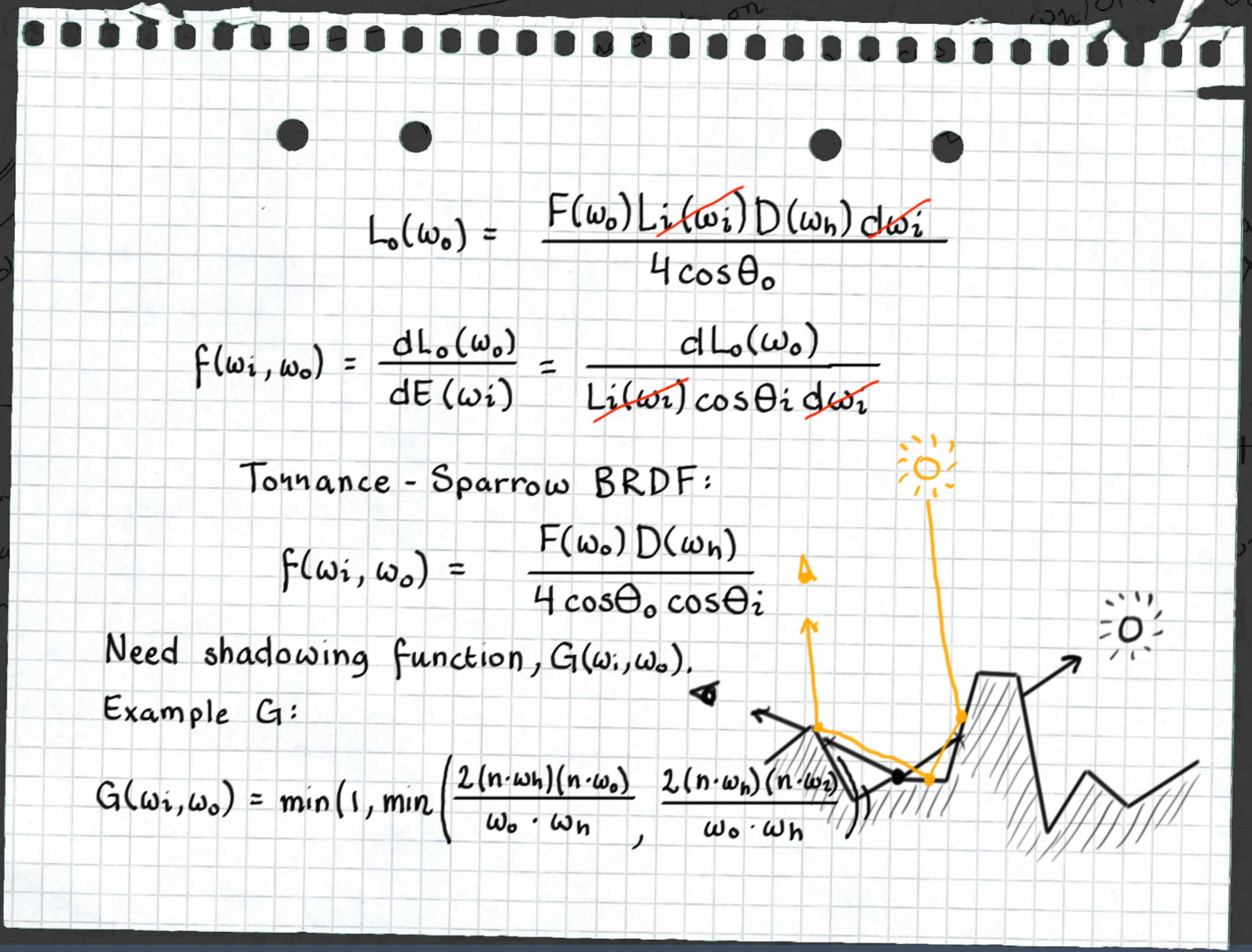
$$L_o(\omega_o) = \frac{d\Phi_o}{d\omega_o \cos\theta_o\, dA} = \frac{F(\omega_o)\, L_i(\omega_i)\, d\omega_i \cos\theta_h\, D(\omega_h)\, d\omega_h\, dA}{d\omega_o \cos\theta_o\, dA}$$

$$L_o(\omega_o) = \frac{F(\omega_o)\, L_i(\omega_i)\, d\omega_i\, \cancel{\cos\theta_h}\, D(\omega_h)\, \frac{\cancel{d\omega_o}}{4\cancel{\cos\theta_h}}\, \cancel{dA}}{\cancel{d\omega_o} \cos\theta_o\, \cancel{dA}}$$

$$= \frac{F(\omega_o)\, L_i(\omega_i)\, D(\omega_h)\, d\omega_i}{4\cos\theta_o}$$

Microfacet theory

- We begin by figuring out how much flux is reflected from one direction.
- Then we find outgoing radiance
- And then we can express the Torrance Sparrow BRDF
- Add shadowing/masking function
- What about interreflection?



$$L_o(\omega_o) = \frac{F(\omega_o)\cancel{L_i(\omega_i)}D(\omega_h)\cancel{d\omega_i}}{4\cos\theta_o}$$

$$f(\omega_i,\omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)} = \frac{dL_o(\omega_o)}{\cancel{L_i(\omega_i)}\cos\theta_i\cancel{d\omega_i}}$$

Torrance - Sparrow BRDF:

$$f(\omega_i,\omega_o) = \frac{F(\omega_o)D(\omega_h)}{4\cos\theta_o\cos\theta_i}$$

Need shadowing function, $G(\omega_i,\omega_o)$.

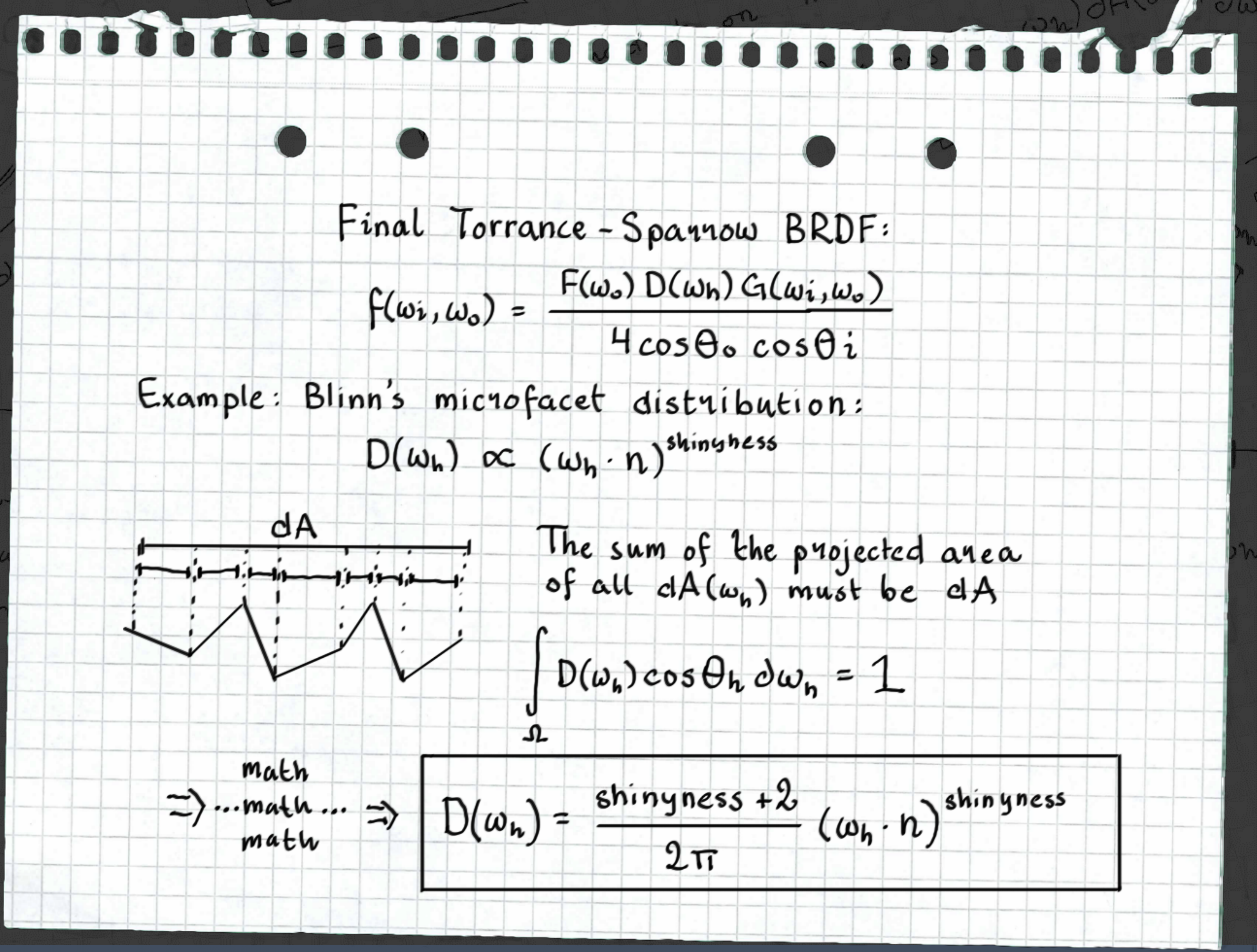
Example G:

$$G(\omega_i,\omega_o) = \min\left(1, \min\left(\frac{2(n\cdot\omega_h)(n\cdot\omega_o)}{\omega_o\cdot\omega_h}, \frac{2(n\cdot\omega_h)(n\cdot\omega_i)}{\omega_o\cdot\omega_h}\right)\right)$$

Microfacet theory

- Final Torrance Sparrow BRDF
  - F, G and D are interchangeable
  - (but facets must be perfect mirrors)

- One possible distribution D, which should be familiar, was suggested by Blinn [1977]



Final Torrance-Sparrow BRDF:

$$f(\omega_i,\omega_o) = \frac{F(\omega_o)\,D(\omega_h)\,G(\omega_i,\omega_o)}{4\cos\theta_o\,\cos\theta_i}$$

Example: Blinn's microfacet distribution:

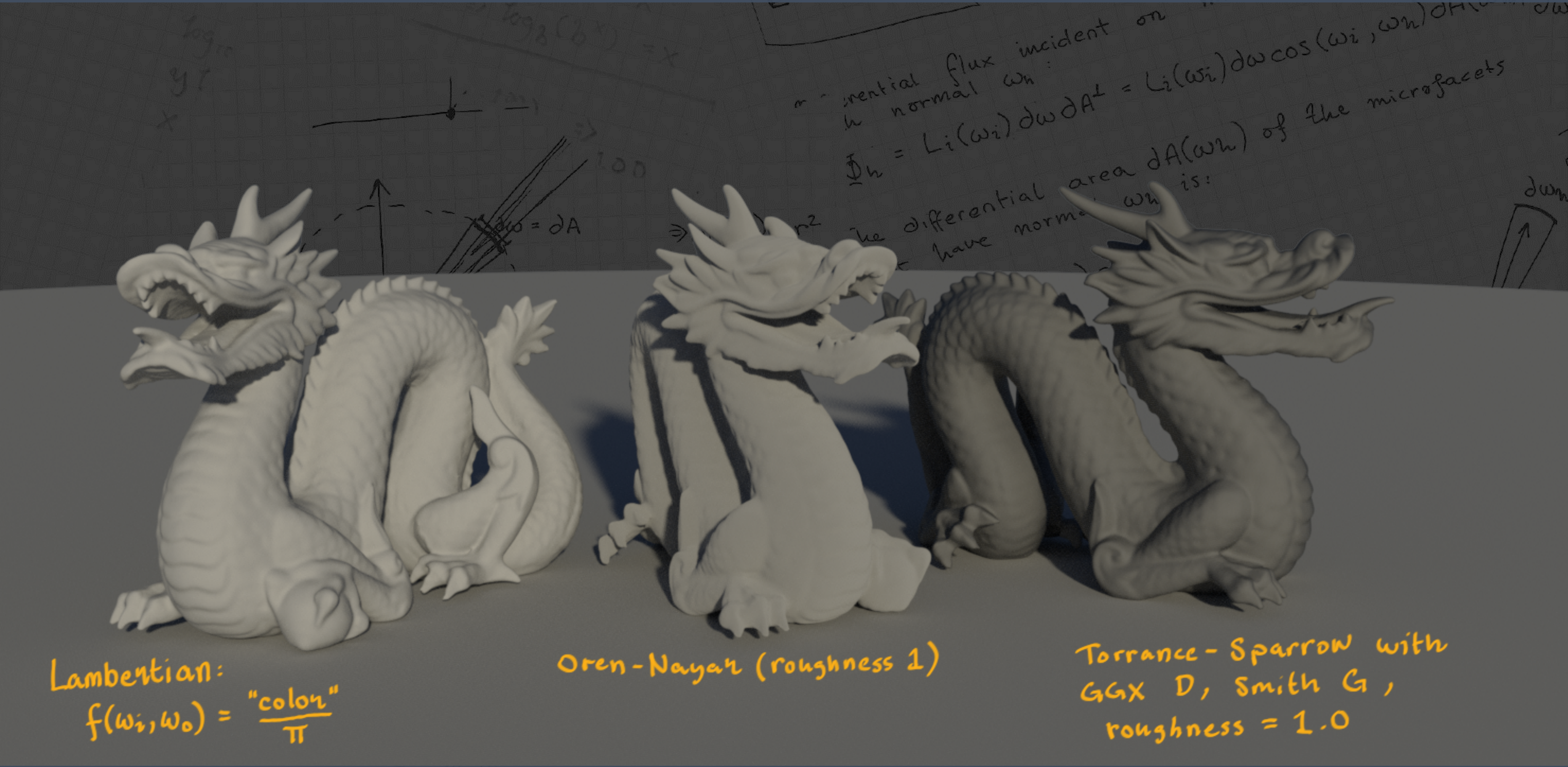
$$D(\omega_h) \propto (\omega_h\cdot n)^{shinyness}$$

The sum of the projected area of all $dA(\omega_h)$ must be dA

$$\int_\Omega D(\omega_h)\cos\theta_h\, d\omega_h = 1$$

$$\Rightarrow \overset{\text{math}}{\underset{\text{math}}{\ldots\text{math}\ldots}} \Rightarrow \boxed{D(\omega_h) = \frac{shinyness+2}{2\pi}(\omega_h\cdot n)^{shinyness}}$$

What about diffuse?

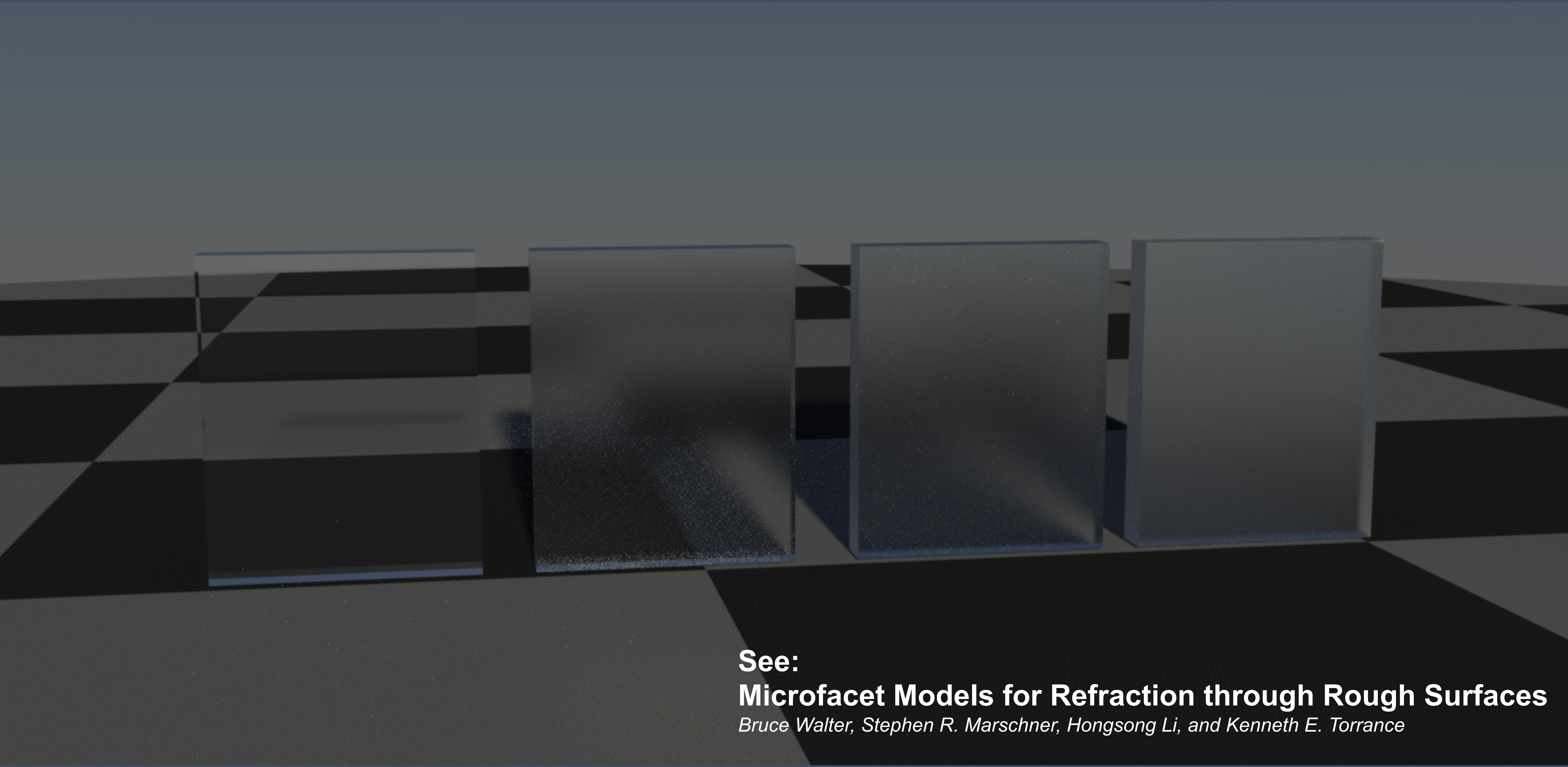


Lambertian:
$f(\omega_i, \omega_o) = \frac{\text{"color"}}{\pi}$

Oren-Nayar (roughness 1)

Torrance-Sparrow with GGX D, Smith G, roughness = 1.0

What else did we skip? – Rough Refractions

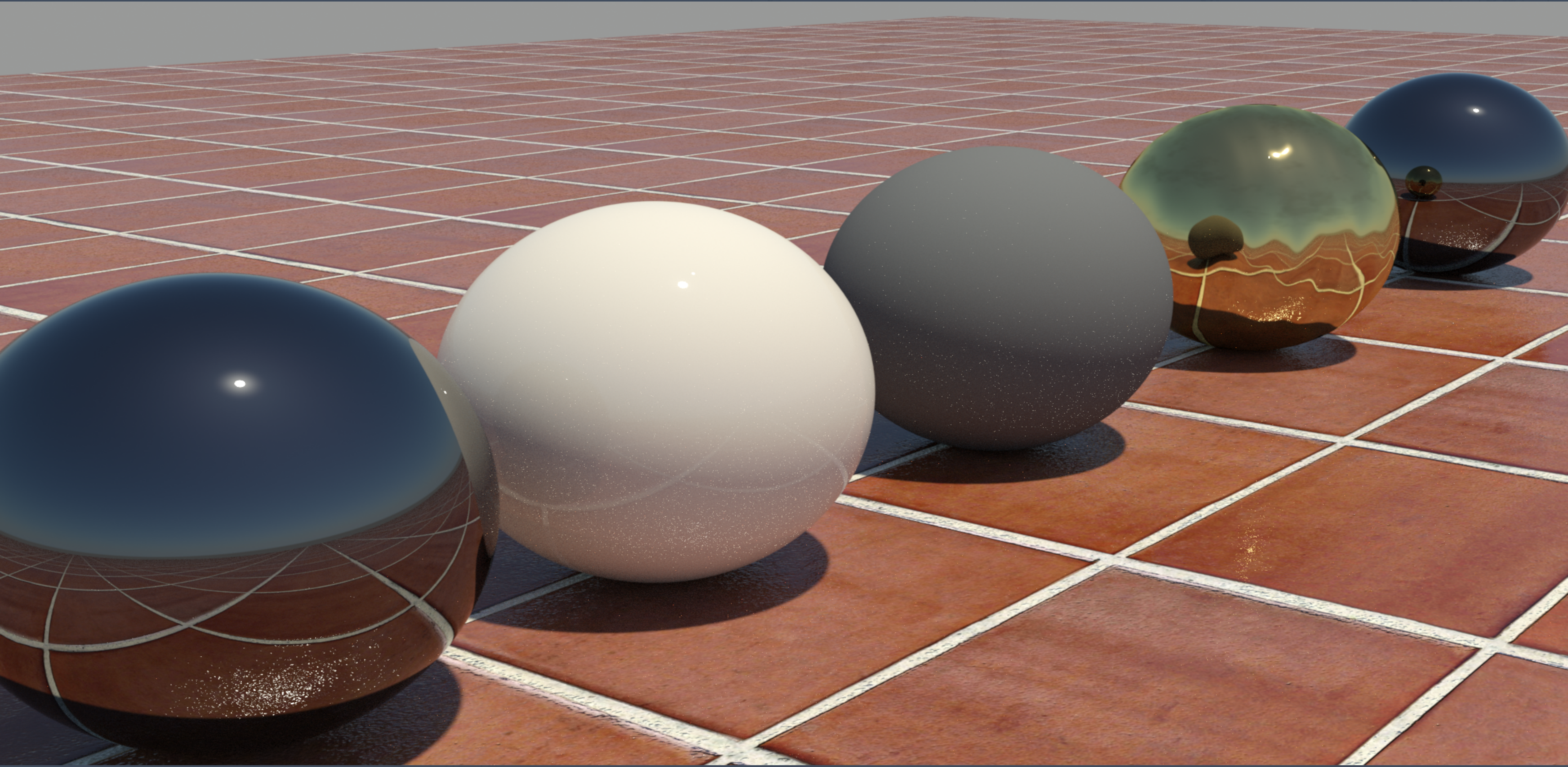


See:

Microfacet Models for Refraction through Rough Surfaces

Bruce Walter, Stephen R. Marschner, Hongsong Li, and Kenneth E. Torrance

What else did we skip? – Complex Materials



What else did we skip? – Subsurface Scattering